INSTITUT D'ASTROPHYSIQUE DE PARIS

Unité mixte de recherche 7095 — *CNRS - Université Pierre et Marie Curie*

Hot Topics in General Relativity and Gravitation

SUMMARY TALK ON GRAVITATION

Luc Blanchet

Gravitation et Cosmologie ($\mathcal{G}\mathbb{R}\varepsilon\mathbb{C}\mathcal{O}$)
Institut d'Astrophysique de Paris

02 aout 2013

Plan of the talk

1. Gravitational waves
   - Experimental
   - Theoretical
2. Experimental gravity
   - Equivalence principle
   - Tests of general relativity
3. Modified gravity theories
   - Massive gravity
   - MOND

GRAVITATIONAL WAVES

Large-scale ground-based laser interferometric detectors

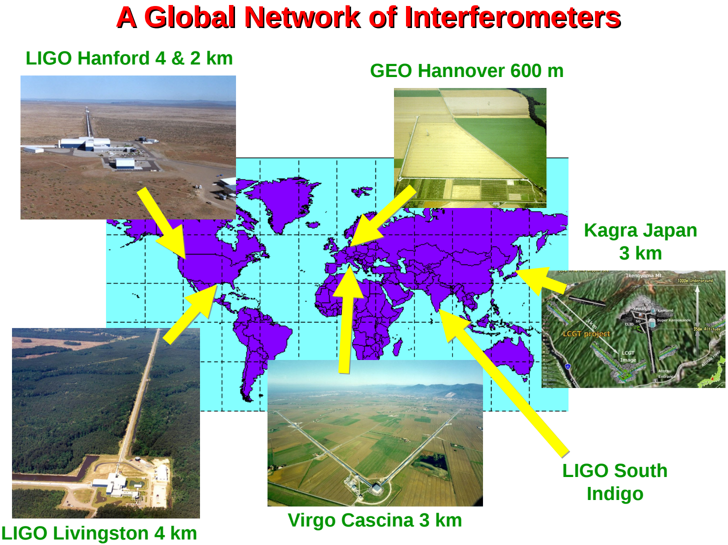
LIGO (L & H)

VIRGO

LIGO and VIRGO will observe the GWs in the high-frequency band

$$10\,\mathrm{Hz} \lesssim f \lesssim 10^3\,\mathrm{Hz}$$

World-wide network of laser interferometric detectors



Status and prospects of current generation of detectors

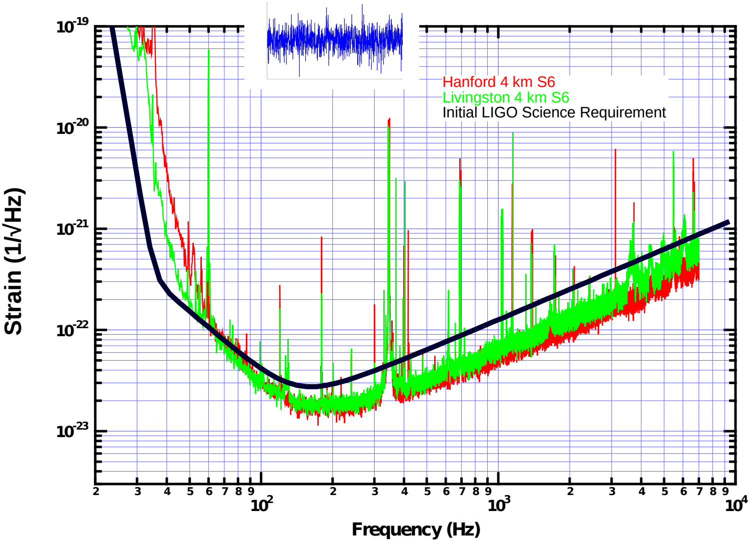
Talks by

- David Blair: New concepts in gravitational wave detection
- Gabriela Gonzalez: Current status and future prospects of GW detectors
- Nary Man: Virgo the French-Italian project for detecting gravitational waves
- Shinji Miyoki: KAGRA large-scale cryogenic gravitational wave telescope

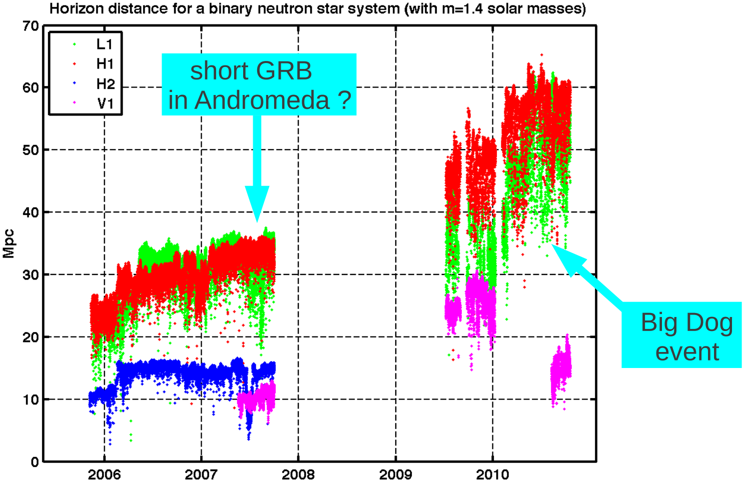
1. GW interferometers are among the most sensitive instruments ever created
    - sensitive to energies $\sim 10^{-32}$ J
    - detect by interferometry tiny displacements $\sim 10^{-18}$ m
    - work close to limits set by the uncertainty principle
2. Like for the search of the Higgs particle
    - well-understood detectors
    - well-understood physics
    - firm predictions
    - hope for surprises

$$\boxed{\text{Prediction of GW [Einstein 1916]} \Longrightarrow \text{Detection of GW [LIGO/VIRGO/}\cdots\text{ 2016] ?}}$$

Noise curve of LIGO detectors (*circa* 2010)

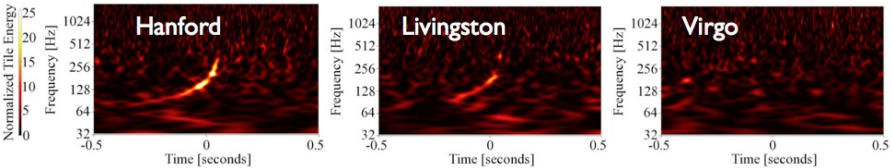


Some interesting events (2005-2011)



Gravitational wave event detected?

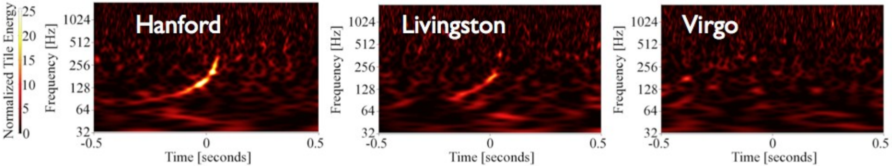
Sept 16, 2010: "Big Dog event"



False alarm rate ~1/7000 yrs!

A case for falling deep into depression

Sept 16, 2010: "Big Dog event"



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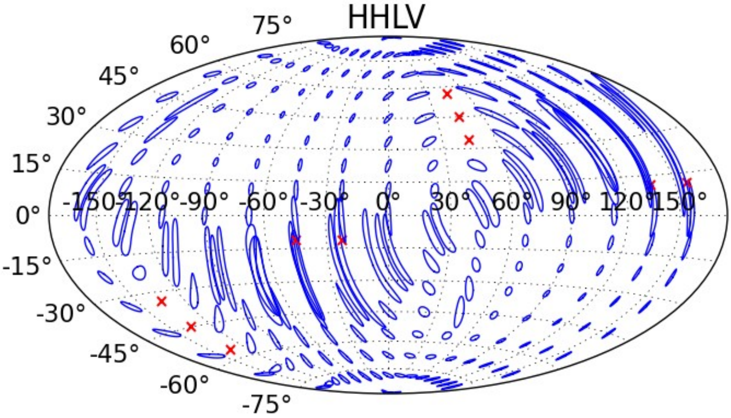
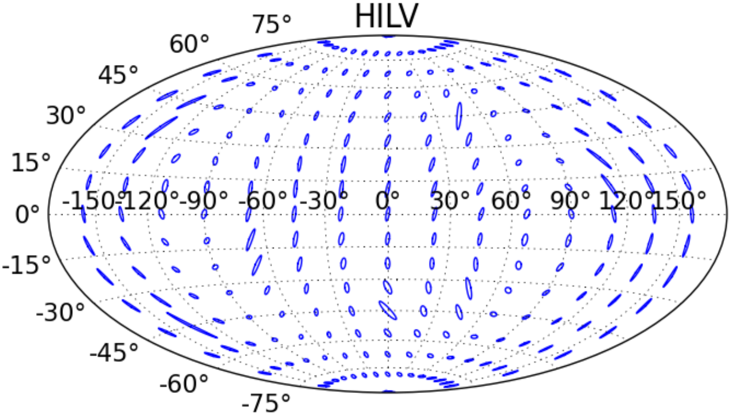
BNS merger localisation: Hanford-Livingston-Virgo



90% localization ellipses for face-on BNS sources @ 160 Mpc

BNS merger localisation: Hanford-Livingston-Virgo-India



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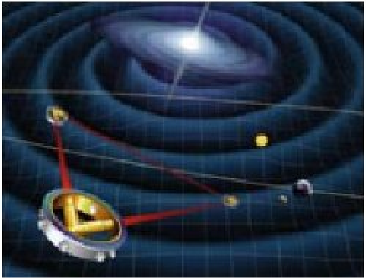
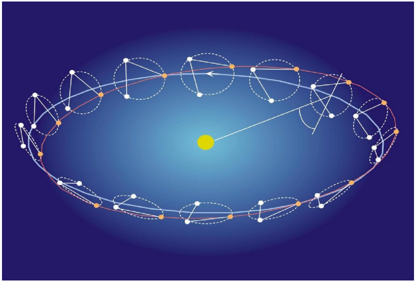
Space-based laser interferometric detector

LISA

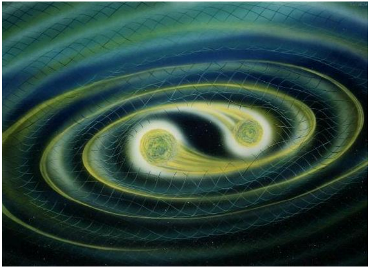




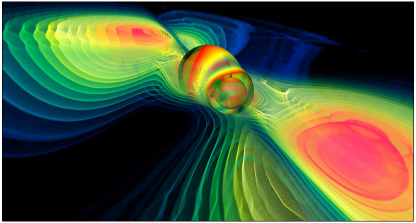
LISA will observe the GWs in the low-frequency band

$$10^{-4}\,\mathrm{Hz} \lesssim f \lesssim 10^{-1}\,\mathrm{Hz}$$

The inspiral and merger of compact binaries



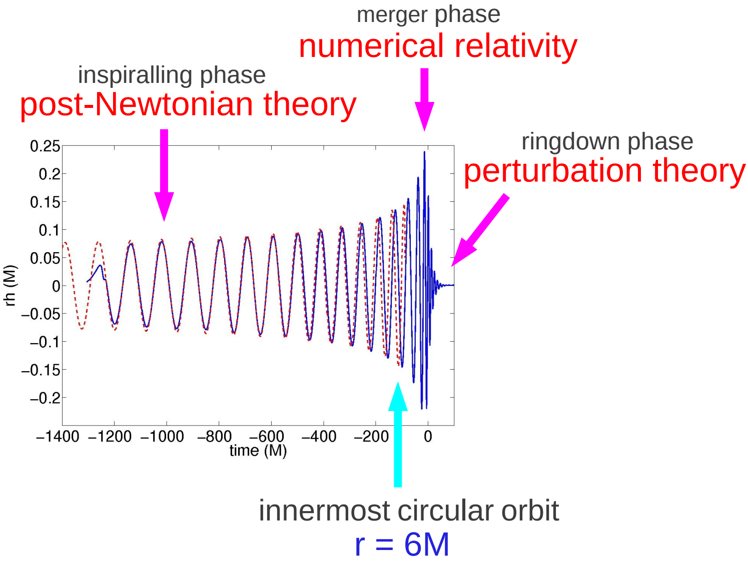
Neutron stars spiral and coalesce



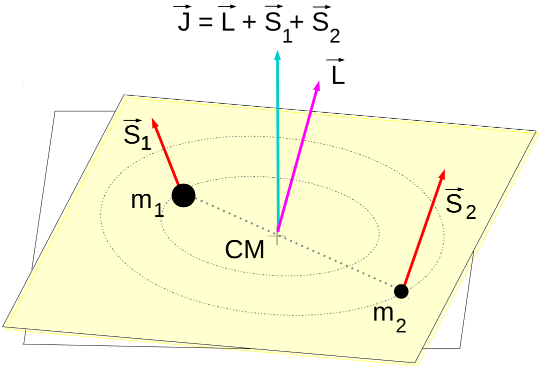
Black holes spiral and coalesce

1. Neutron star ($M = 1.4\,M_\odot$) events will be detected by ground-based detectors LIGO/VIRGO/$\cdots$
2. Stellar size black hole ($5\,M_\odot \lesssim M \lesssim 20\,M_\odot$) events will also be detected by ground-based detectors
3. Supermassive black hole ($10^5\,M_\odot \lesssim M \lesssim 10^8\,M_\odot$) events will be detected by the space-based detector LISA

The gravitational chirp of compact binaries



Binary System of black holes with spins



Talk by Sylvain Marsat

Recent results on spin-orbit effects at the next-to-next-to-leading order

- $1.5\text{PN} + 2.5\text{PN} + 3.5\text{PN}$ in the equations of motion
- $1.5\text{PN} + 2.5\text{PN} + 3.5\text{PN} + 4\text{PN}^{\text{tail}}$ in the orbital phase

Typical post-Newtonian coefficient

$$\frac{2388425}{3024}\pi$$

Gravitational waves related insights

Talk by Brahim Lamine

- Magnetosphere of neutron stars above critical magnetic field $B_c \sim 4\,10^{13}$ G contains virtual electron-positron pairs described at the one-loop level by an effective action [Euler & Heisenberg 1939]
- Induces a magnetic susceptibility of the vacuum and a modification of the magnetic field by "quantum vacuum friction"
- Yields an additional energy loss mechanism which changes the spin-down evolution of highly magnetized pulsars

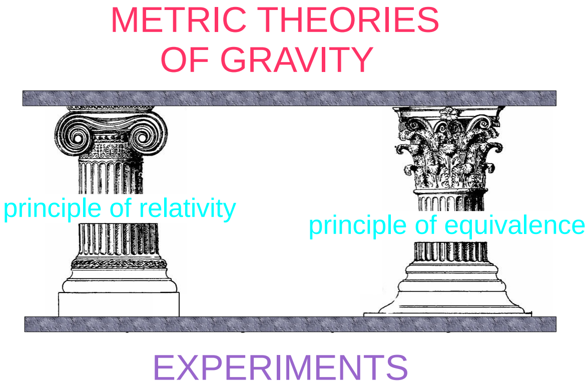
Talk by Mikhail Katanaev

- Proof that the Schwarzschild solution satisfies the Einstein field equations with singular point-like source in a generalized sense [a]

[a] Russian people call this in that way after [Gel'fand & Shilov 1958]; French people rather say in the sense of Schwartz's distributions [Schwartz 1958]

EXPERIMENTAL GRAVITY

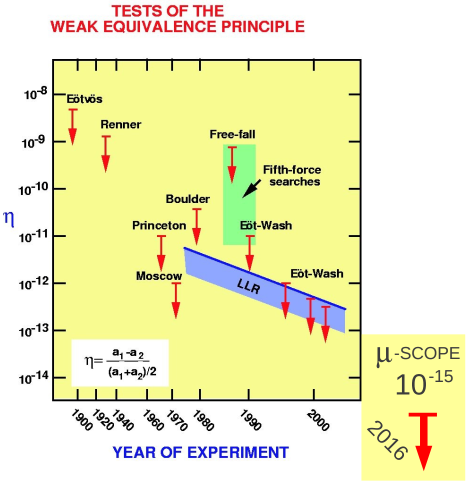
Foundations of gravity theories



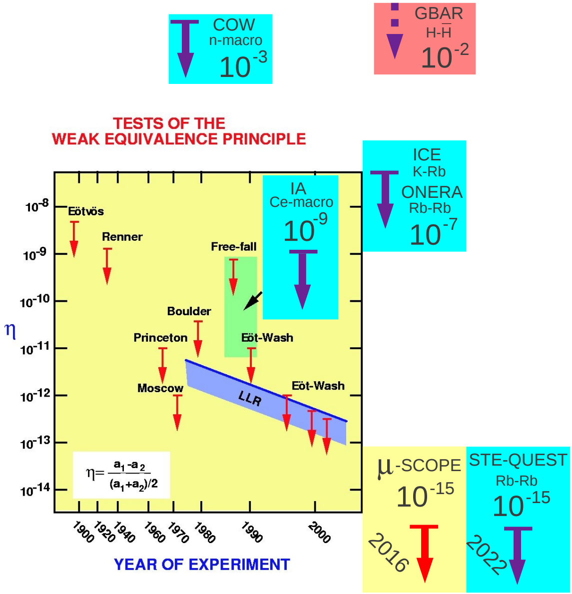
- Michelson-Morley experiments
- Hughes-Drever experiments
- · · ·

- Torsion-balance experiments
- Gravitational redshift experiments
- · · ·

Experimental limits on the weak equivalence principle

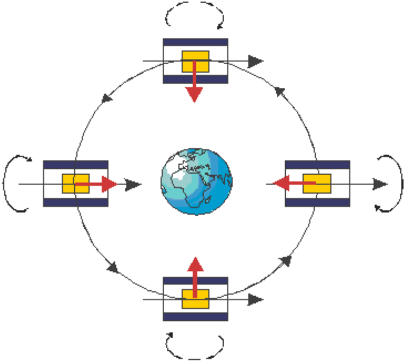


Experimental limits on the WEP (quantum tests)

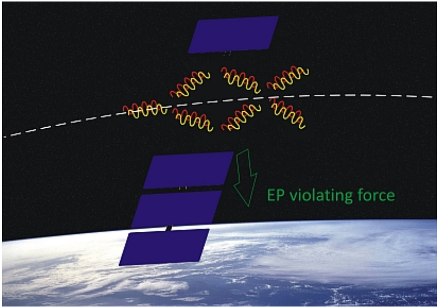


Space experiments testing the WEP

μ-SCOPE experiment

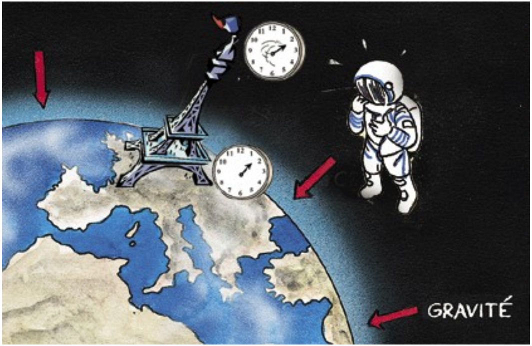


STE-Quest experiment



Expected accuracy 10^{-15}

Pharao-ACES experiment to test the gravitational redshift



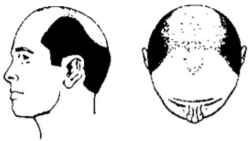
Expected accuracy 10^{-6}

Talk by Luc Blanchet

Recent claims [Müller, Peters & Chu 2010] that it is possible to test the gravitational redshift with high accuracy $\sim 10^{-9}$ by atom interferometry experiments performed in the laboratory are incorrect

Testing the no-hair theorem for black-holes

"Théorème de la calvitie des trous noirs"



The multipole moments Q_ℓ and J_ℓ of the Kerr geometry extracted from the expansion of the metric at spatial infinity obey [Hansen 1974]

$$Q_\ell + \mathrm{i} J_\ell = M \, (\mathrm{i}a)^\ell$$

Talk by Clifford Will

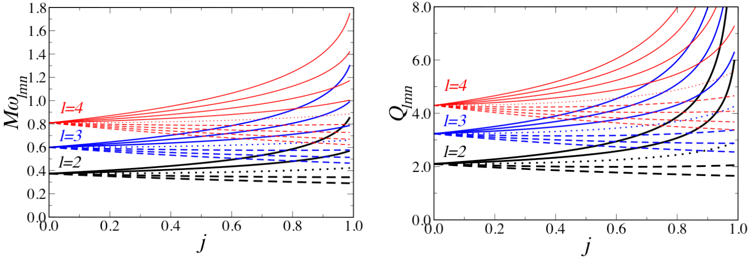
Counting hair of black holes is possible:

1. Using gravitational waves
   - Extreme mass ratio inspiral (EMRI)
   - Ringdown radiation after merging of two BHs
2. With the BH at the galactic center

Counting hair with ringdown radiation

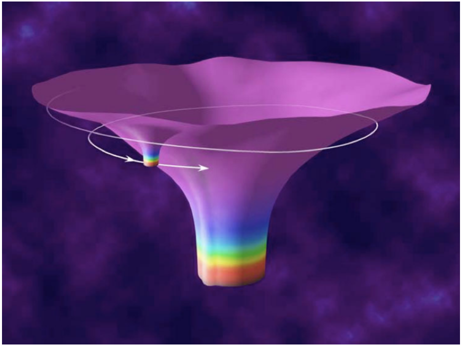
- Merger of two black holes produces distorted BHs who radiate "ringdown" waves to shed hair
- Frequency modes of ringdown radiation (*e.g.* [Berti, Cardoso & Will 2006])

$$\omega = \omega_{\ell mn}\left[1 + \frac{\mathrm{i}\pi}{2Q_{\ell mn}}\right]$$



$$j = \frac{J}{M^2}$$

Counting hair using EMRI

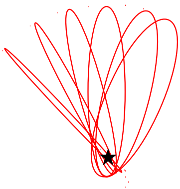


- Neutron star or stellar-size black hole follows a highly relativistic orbit around a supermassive black hole
- Observations of EMRIs by LISA permit to count hair of the central BH, verifying the no-hair theorem

Counting hair at the galactic center (SgrA*)

Usual relativistic effects on the orbit

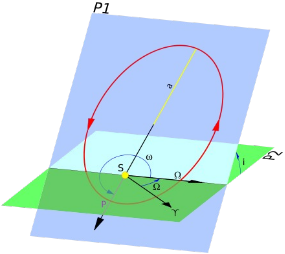
- Perihelion advance
- Redshift and Doppler shifts
- Shapiro delays



Talk by Clifford Will

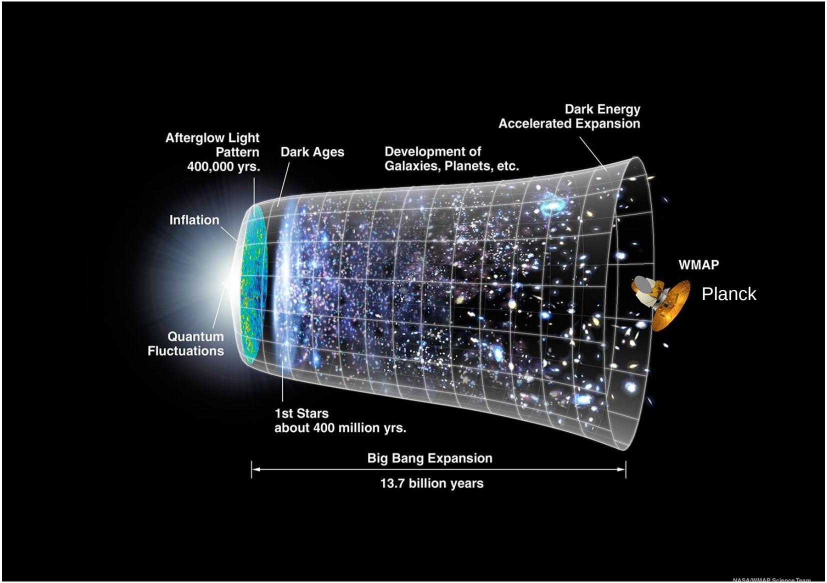
Precession of the orbital plane produced by frame dragging (J) and quadrupole moment (Q) as no-hair test for SgrA*

$$\begin{aligned} A_M &= 6\pi \frac{M}{\bar{a}(1-e^2)} \\ A_J &= 4\pi \frac{J}{M^2} \left(\frac{M}{\bar{a}(1-e^2)}\right)^{3/2} \\ A_Q &= 3\pi \frac{Q}{M^3} \left(\frac{M}{\bar{a}(1-e^2)}\right)^{2} \end{aligned}$$



MODIFIED GRAVITY THEORIES

Motivations for modified gravity theories



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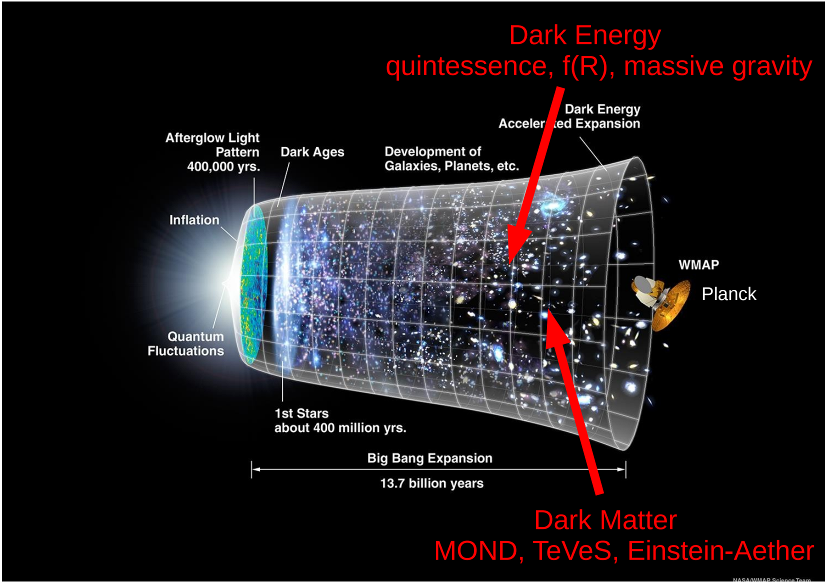

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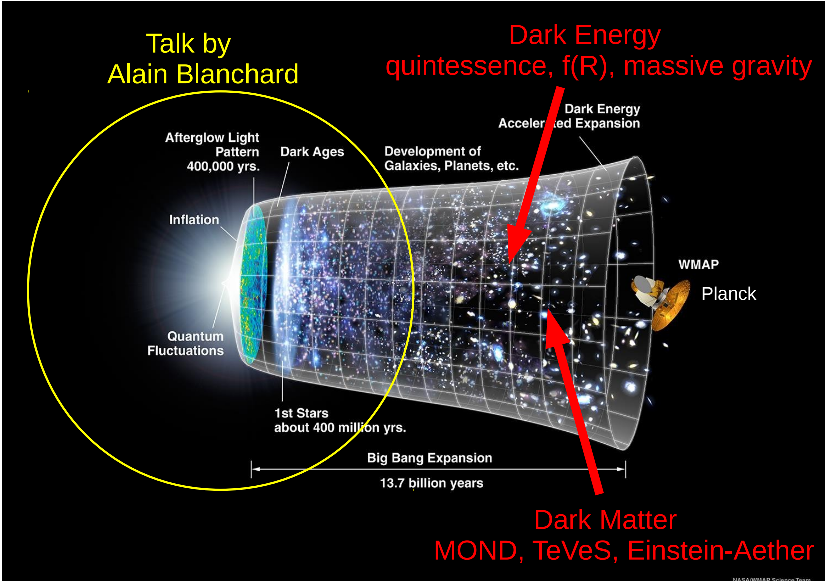
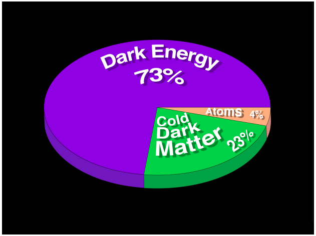


Problem of the fundamental constituents of the Universe



Λ-CDM assumes General Relativity is the correct theory of gravity
This poses the problem of the unknown constituents of the Universe

1. No known particle in the standard model of particle physics could be the particle of dark matter
2. Extensions of the standard model of particle physics provide well-motivated but yet to be discovered candidates
3. The cosmological constant Λ does not have the right value when interpreted as a vacuum energy associated with the fluctuations of the gravitational field

History of massive gravity theories

1. Action of Fierz & Pauli [1939]: Unique linear theory without ghosts

$$S_{\mathsf{FP}} = \frac{1}{16\pi}\int \mathrm{d}^4x \Big[\underbrace{\partial_\mu h_{\nu\rho}\partial^\mu \overline{h}^{\nu\rho} - H_\mu H^\mu}_{\substack{\text{Einstein-Hilbert action}\\ \text{for } g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}}} + \underbrace{m_{\mathrm{g}}^2\big(h_{\mu\nu}h^{\mu\nu} - h^2\big)}_{\text{mass term}}\Big]$$

2. Discontinuity of Van Dam, Veltman & Zhakharov [1970]: Massless limit of massive gravity differs from GR and is invalidated in the solar system

$$\gamma_{\mathsf{PPN}} = \frac{1}{2} \quad (?)$$

3. Vainshtein [1972] mechanism: Nonlinear terms restaure the continuity of the massless limit and GR is recovered for

$$r < r_{\mathsf{Vainshtein}} = \left(\frac{M}{m_{\mathrm{g}}^4}\right)^{1/5}$$

4. Boulware-Deser [1972] ghost: Instability appears at quadratic order due to 6 degrees of freedom for the massive spin-2 field instead of 5

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Nonlinear ghost-free massive gravity theory

[de Rham & Gabadadze 2010; de Rham, Gabadadze & Tolley 2010]

1. We covariantize the FP action by introducing four scalar Stukelberg fields ϕ^a $(a = 0, 1, 2, 3)$ and defining the perturbation of the metric $g_{\mu\nu}$ as

$$H_{\mu\nu} = g_{\mu\nu} - \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$$

2. The resulting theory is a tensor-multi-scalar theory. Alternatively this is also a bimetric theory since

$$f_{\mu\nu} = \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$$

   plays the role of an auxiliary metric

3. The mass term is constructed purely algebraically [1] as a non-linear interaction between the two metrics $g_{\mu\nu}$ and $f_{\mu\nu}$

[1] This point is in constrast with recent MOND-like bimetric theories where the interaction term is constructed as a differential non-linear interaction between the two metrics [Talks by M. Milgrom and L. Blanchet]

Nonlinear ghost-free massive gravity theory

[de Rham & Gabadadze 2010; de Rham, Gabadadze & Tolley 2010]

The theory is defined non-perturbatively as

$$S = \frac{1}{16\pi}\int \mathrm{d}^4x\sqrt{-g}\left[R[g] + \overbrace{m_\mathrm{g}^2\Big(L_2 + \alpha_3 L_3 + \alpha_4 L_4\Big)}^{\text{mass term}}\right]$$

where we define the matrix $\mathcal{K} \equiv \mathbb{1} - \sqrt{g^{-1}f}$ and pose

$$\begin{aligned}
L_2 &= \frac{1}{2}\Big[(\mathrm{Tr}\mathcal{K})^2 - \mathrm{Tr}(\mathcal{K}^2)\Big]\\
L_3 &= \frac{1}{6}\Big[(\mathrm{Tr}\mathcal{K})^3 - 3\mathrm{Tr}\mathcal{K}\mathrm{Tr}(\mathcal{K}^2) + 2\mathrm{Tr}(\mathcal{K}^3)\Big]\\
L_4 &= \frac{1}{24}\Big[(\mathrm{Tr}\mathcal{K})^4 - 6(\mathrm{Tr}\mathcal{K})^2\mathrm{Tr}(\mathcal{K}^2) + 8\mathrm{Tr}\mathcal{K}\mathrm{Tr}(\mathcal{K}^3) - 6\mathrm{Tr}(\mathcal{K}^4)\Big]
\end{aligned}$$

- Q1: Can we add an Einstein-Hilbert term for the metric $f_{\mu\nu}$ as well?
- Q2: Can this construction be generalized in D dimensions?

Applications of massive gravity theory

Talks by

1. Kei-Ichi Maeda: Dynamics of bigravity and general relativity
2. Claudia de Rham: Modified Gravity and Binary Pulsar Tests
3. Shinji Mukohyama: Massive gravity and cosmology

- Q3: With GWs emitted by inspiralling compact binaries one can bound the mass of the graviton m_g. The argument was given at a time where there was no consistent massive gravity theory [Will 1999]. Do you think with the new theory the same bound can be proved rigourously?

Dark matter or modified gravity? [Le Verrier 1846]

"Modified gravity"

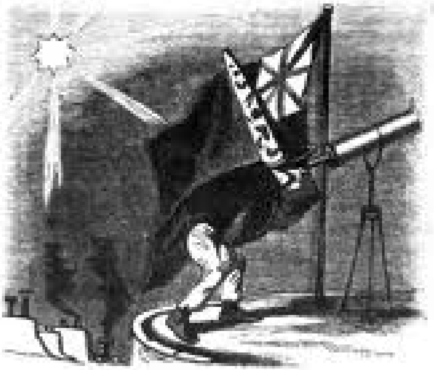
Anomalous orbital precession of the planet Mercury
finally explained by general relativity

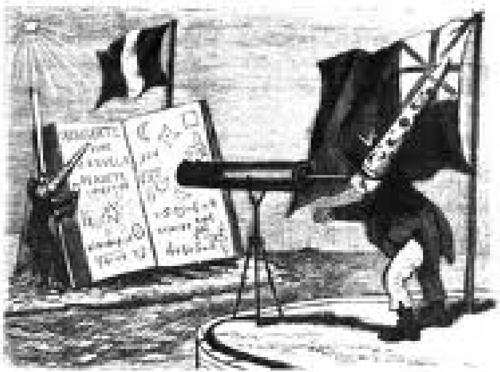
Dark matter or modified gravity? [Le Verrier 1845]

"Dark matter"

Discovery of the planet Neptune from the
Newtonian perturbations induced on the planet Uranus

Planet Neptune also discovered by Adams [unpublished]





Phenomenology of galaxy dynamics (or why MOND is still relevant)?

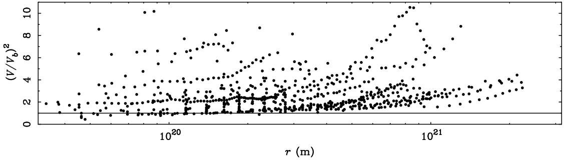
MOND was introduced 30 years ago [Milgrom 1983]

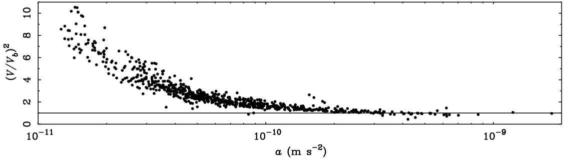
- It is still relevant to galaxy dynamics despites its strong falsifiability
- It still represents a complete mystery for why it works so well

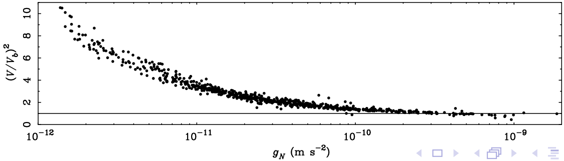
Talk by Benoit Famaey

1. Fine-tuned relative distribution of baryons and DM in galaxies
2. Several *a priori* independent occurences of the acceleration scale a_0
   - Transition acceleration where the dynamical effects of DM appear
   - Defines a critical baryonic surface density a_0/G for disk stability
   - Defines the typical DM halo surface density $\rho_0 r_0^3$ at center
3. Dwarf galaxies paucity and geometry

Phenomenology of galaxy dynamics (or why MOND is still relevant)?







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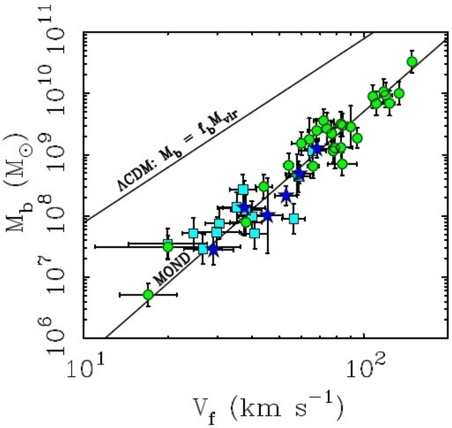


Talk by Benoit Famaey

$$V_{\rm f} \approx \left(G\, M_{\rm b}\, a_0\right)^{1/4} \quad \text{where} \quad a_0 \approx 1.2\times 10^{-10}\text{m/s}^2$$

Bimetric MOND gravity (BIMOND)

Talk by Moti Milgrom

1. MOND is a theory of dynamics (either gravity or inertia) involving a new fundamental constant a_0 (besides G, c, $\hbar$)
   - Newtonian limit: $a_0 \to 0$
   - MOND limit: $a_0 \to \infty$ and $G \to 0$ with $\Omega_0 = G a_0$ fixed
2. MOND relativistic extensions of GR:
   - Tensor-vector-scalar theory [Bekenstein 2004, Sanders 2005]
   - Non-canonical Einstein-Æther theories [Zlosnik *et al.* 2007, Halle *et al.* 2008]
   - $\cdots$

Bimetric MOND gravity (BIMOND)

1. BIMOND is a particular relativistic theory reproducing MOND in the non-relativistic approximation

$$\begin{aligned} S_{\mathsf{BIMOND}} &= \frac{1}{16\pi}\int \mathrm{d}^4x\left[(-g)^{1/2}R[g] + (-\hat{g})^{1/2}R[\hat{g}] + \overbrace{2a_0^2(g\hat{g})^{1/4}F\left[\frac{Z}{a_0^2}\right]}^{\text{interaction term}}\right] \\ &+ \underbrace{S_{\mathsf{m}}[g_{\mu\nu},\psi_i]}_{\text{ordinary matter}} + \underbrace{\hat{S}_{\mathsf{m}}[\hat{g}_{\mu\nu},\chi_i]}_{\text{twin matter}} \end{aligned}$$

2. The interaction term between the two metrics $g_{\mu\nu}$ and $\hat{g}_{\mu\nu}$ is constructed out of the difference of Christoffel symbols $C^\lambda_{\mu\nu} \equiv \Gamma^\lambda_{\mu\nu} - \hat{\Gamma}^\lambda_{\mu\nu}$ (which is a tensor)

$$\Psi = \frac{1}{2}\left(g^{\mu\nu} + \hat{g}^{\mu\nu}\right)\left[C^\gamma_{\mu\lambda}C^\lambda_{\nu\gamma} - C^\gamma_{\mu\nu}C^\lambda_{\lambda\gamma}\right]$$

The dielectric analogy of MOND

- Gauss' equation is modified by the polarization inside dielectric materials

$$\nabla \cdot \Big[\underbrace{(1+\chi_e)\boldsymbol{E}}_{\boldsymbol{D}\ \text{field}} \Big] = \frac{\rho_e}{\varepsilon_0} \qquad \Longleftrightarrow \qquad \nabla \cdot \boldsymbol{E} = \frac{\rho_e + \rho_e^{\text{polar}}}{\varepsilon_0}$$

- MOND appears as Poisson's equation modified by gravitational polarization

$$\nabla \cdot \Big[\underbrace{\mu\left(\frac{g}{a_0}\right)}_{\text{MOND function}} \boldsymbol{g} \Big] = -4\pi\, G\, \rho_{\mathrm{b}} \qquad \Longleftrightarrow \qquad \nabla \cdot \boldsymbol{g} = -4\pi\, G \left(\rho_{\mathrm{b}} + \underbrace{\rho^{\text{polar}}}_{\text{dark matter}} \right)$$

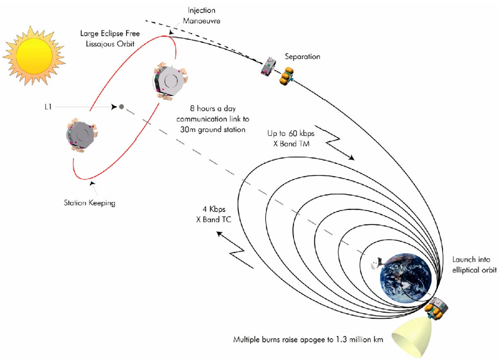
Talk by Luc Blanchet

Three models reproducing MOND and based on the notion of dipolar dark matter

- Newtonian model (non-viable)
- Relativistic model in standard GR [Blanchet & Le Tiec 2008, 2009]
- Bimetric model [in progress]

Testing MOND with LISA-Pathfinder?

- It has been suggested that LPF may visit the saddle point of the Earth-Sun system and test gravity in the weak field limit



- However the MOND sphere around the saddle point (for which $g \lesssim a_0$) is only a few meters in diameter
- LPF will only be able to test the transition between the Newtonian and MOND regimes in the specific TeVeS theory