Towards event-by-event studies of the ultrahigh-energy cosmic-ray composition

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Motivation

What can we learn from GZK-studies,
if the cutoff would be observed?

Astrophysics	Particle physics
sources:	interactions at $E_{cm} \gtrsim 300$ TeV:
accelaration mechanism,	cross section,
local backgrounds	inelasticity,
space:	multiplicity,
γ-background,	P_T-distribution,
magnetic fileds	new physics phenomena?

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chemical composition

Plan

- The main idea: event-by-event analysis
- The probability of a given particle (p, γ, ...) to be a primary of a given observed event
- Chemical composition from observed events with "estimated" primary type
- Several Examples: highest energy AGASA and Yakutsk events, low energy Yakutsk events
- What else?

Event-by-event analysis

It is obviously important for studies of:

- highest energy tail
- evolution of chemical compoistion with energy
- global anisotropy (direction-dependent composition)?

Problems with poor statistics:

- just several parameters of EAS are measured
- significant fluctuations
- degeneracy: similar values for different primaries
- azimuth angle dependence

Interesting questions: a given event

For a given observed event (several parameters of EAS are measured):

- conservative study (negative knowledge): what is the probability, that it could not be initiated by the primary A?
- (positive knowledge): what is the probability, that it could be initiated by a primary of a given type A_i (say, $A_i = p, \gamma, \ldots,$ Fe)?
- ...

What is the primary of an observed event?

energy-related parameters of EAS ($S_{600}, \ldots$) — E-parameters

composition-related parameters of EAS ($\rho^{\mu}_{1000}, \ldots$) — c-parameters

Both parameters are reconstructed with some errors

The probability distribution that the primary particle which produced an actual shower with the observed E-parameters equal to $E_{\rm obs}$ would rather produce a shower with these parameters equal to $E_{\rm rec}$:

$$g_E(E_{\rm rec}, E_{\rm obs})$$

The probability distribution that a shower with measured c-parameters equal to $\mathbf{c}$ could produce detector readings corresponding to $\mathbf{c}'$:

$$g_c(\mathbf{c}', \mathbf{c}).$$

Steps

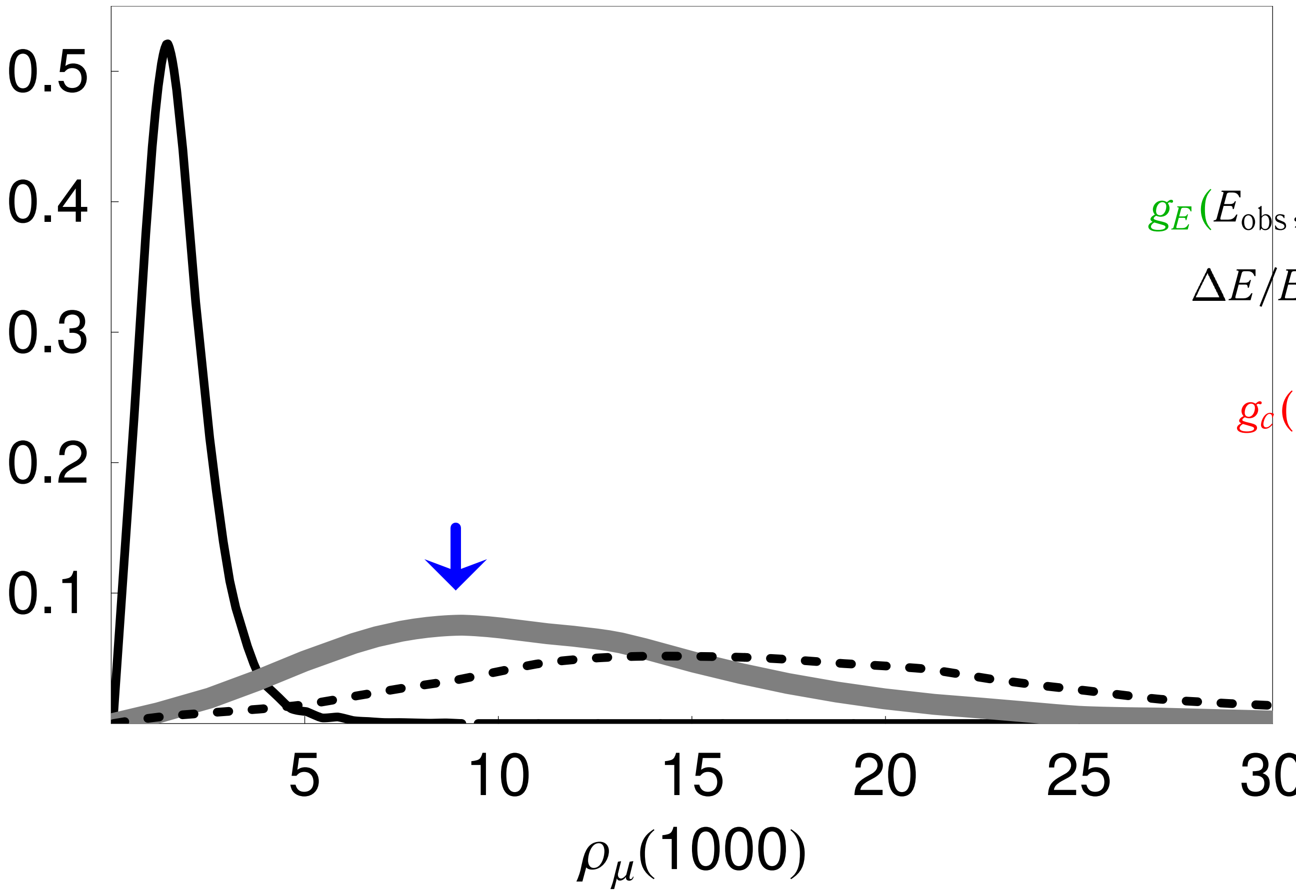
1. for each primary one generates a library of $\mathcal{N}$ simulated showers : the same direction, $E_s \sim E_{\rm obs}$, e.g. $0.5E_{\rm obs} < E_s < 2E_{\rm obs}$
2. following the experimental procedure for each event one finds $E_{\rm rec}$ (e.g., $S_{600}^{\rm rec}$)
3. one assigns to each simulated shower a weight $w_1 = g_E(E_{\rm obs}, E_{\rm rec})$
4. one assigns to each simulated shower an additional weight $w_2 = (E_s/E_{\rm obs})^\alpha$ to mimic the real power-law spectrum

Output:
The distribution of the c-parameters for the showers consistent with the real one by E-parameters is given by

$$f_A(\mathbf{c}) = \frac{1}{\mathcal{N}} \sum_i^{\mathcal{N}} g_c(\mathbf{c}, \mathbf{c}_{iA}) w_{1,iA} w_{2,iA}$$

Example

The highest energy AGASA event $2.46 \cdot 10^{20}$ eV: $\quad c = \rho_\mu(1000)$



$$g_E(E_{\rm obs}, E_{\rm rec}) = \frac{e^{-\frac{log^2(E_{\rm rec}/E_{\rm obs})}{2\sigma_l^2}}}{\sqrt{2\pi}\sigma_l}$$

$$\Delta E/E = 0.25 \to \sigma_l = 0.104$$

$$g_c(c', c) = \frac{e^{-\frac{(c-c')^2}{2\sigma_c^2}}}{\int_0^\infty e^{-\frac{(c-c')^2}{2\sigma_c^2}} dc}$$

$$\sigma_c = 0.4c'$$

Distributions of muon densities f_A of simulated events: thin dark line, $A = \gamma$; thick gray line, $A = p$; dashed line, $A =$Fe.

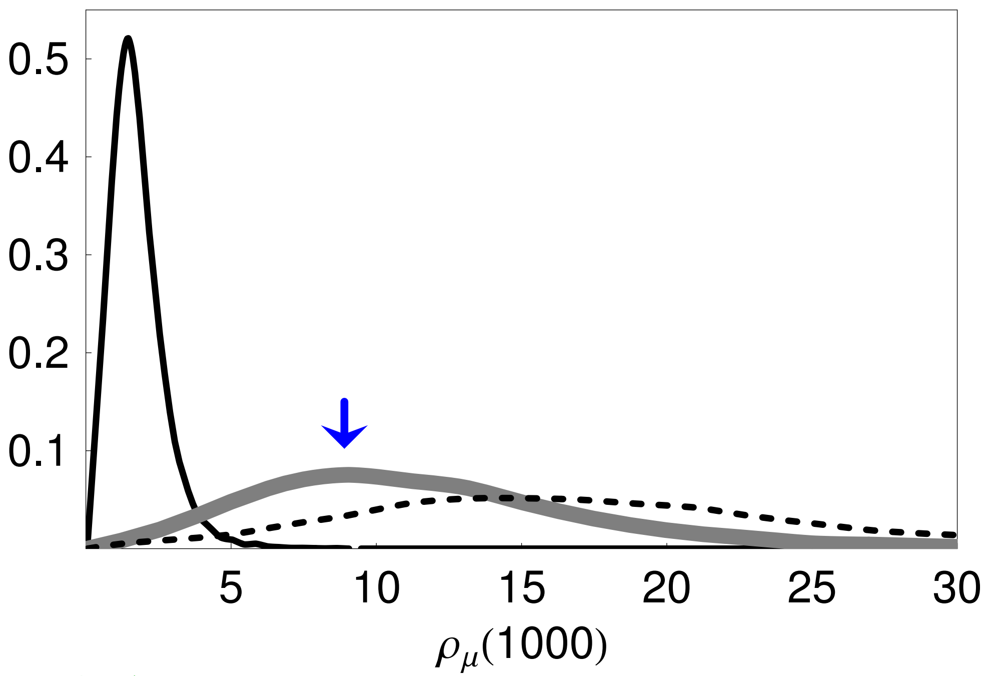
Results

If the event is unlikely being initiated by the primary A, one can estimate the probability of it could be initiated by the primary A:

$$p_A = F_A(\mathbf{c}_{\rm obs}) \equiv \int\limits_{f_A(\mathbf{c}) \le f_A(\mathbf{c}_{\rm obs})} f_A(\mathbf{c}) d\mathbf{c}$$

one can test the hypothesis that the primary was either A_1 or A_2.

Then $p_{A_1} + p_{A_2} = 1$ and



$$p_{A_k} = \frac{f_{A_k}(\mathbf{c}_{\rm obs})}{f_{A_1}(\mathbf{c}_{\rm obs}) + f_{A_2}(\mathbf{c}_{\rm obs})} \ , \quad k = 1, 2$$

Interesting questions: combined set

Chemical composition of UHECR within a given energy interval $E_{min} < E < E_{max} (\equiv \mathcal{E})$ (within a given solid angle, etc.):

- (negative knowledge): the upper limit on the primary A
- (positive knowledge): what is the most probable chemical composition (the set of possible primaries is fixed)?
- ...

Procedure to estimate the chemical composition

1. one selects the set of N observed events (E- , c-parameters have been measured) — experimental cuts ($z < 45^\circ$), quality cuts,...
2. for each event j one compares the parameters of simulated EAS of various primaries to parameters of observed EAS and estimates the probabilities $p_i(j)$ that it could (not) be initiated by a primary A_i of energy within $\mathcal{E}$
3. $p_i(j)$ enables one to estimate the probability that among N events n_i were (not) initiated by primary A_i — combinatorics
4. one estimates the most probable chemical composition ϵ_i or set the upper bound on a primary A_i, which are consistent with selected set of observed events — likelihood
5. one takes into account possible corrections because of cuts on initial set, detector acceptance, etc. — lost events

The upper limit on a fraction of primary A

1. Input

set of N observed events with energies $\mathcal{E}$,

$p_A^{(+)j}$, $p_{\overline{A}}^{(+)j}$, generally $p_A^{(+)j} + p_{\overline{A}}^{(-)j} \neq 1$

Steps

2. Probability $\mathcal{P}(n_1, n_2)$: among N observed events, n_1 initiated by A with $\mathcal{E}$ and n_2 initiated by $\overline{A}$ with $\mathcal{E}$:

The probability to have i_1-th, . . . , i_{n_1}-th events ($i_1 < \cdots < i_{n_1}$) induced by A with $\mathcal{E}$ and k_1-th, . . . , k_{n_2}-th events ($k_1 < \cdots < k_{n_2}$, $i_j \neq k_l$) induced by $\overline{A}$ with $\mathcal{E}$:

$$\mathcal{P}\left(\{i_j\}, \{k_l\}\right) = \prod_{i_j} p_A^{(+)i_j} \prod_{k_l} p_{\overline{A}}^{(+)k_l} \prod_{m_n \neq i_j, k_l} \left(1 - p_A^{(+)m_n} - p_{\overline{A}}^{(+)m_n}\right) ,$$

To calculate $\mathcal{P}(n_1, n_2)$ one sums over all subsets $(\{i_j\}, \{k_l\})$

$$\mathcal{P}(n_1, n_2) = \sum_{\substack{i_1 < i_2 < \cdots < i_{n_1} \\ k_1 < k_2 < \cdots < k_{n_2}}, i_j \neq k_l} \mathcal{P}\left(\{i_j\}, \{k_l\}\right) , \qquad 1 \leq i_j, k_l, m_n \leq N$$

The upper limit on a fraction of primary A

Let ϵ_A be the fraction of A in $\mathcal{E}$

3. Let $\mathcal{P}(\epsilon_A)$ be the probability that the observed results are reproduced for a given ϵ_A. Hence

$$\mathcal{P}(\epsilon_A) = \sum_{n_1,n_2}^{n_1+n_2 \le N} \mathcal{P}(n_1, n_2)\epsilon_A^{n_1} (1 - \epsilon_A)^{n_2}$$

4. Upper limit on ϵ_A at a given confidence level ξ:

$$P(\epsilon_A) \ge 1 - \xi$$

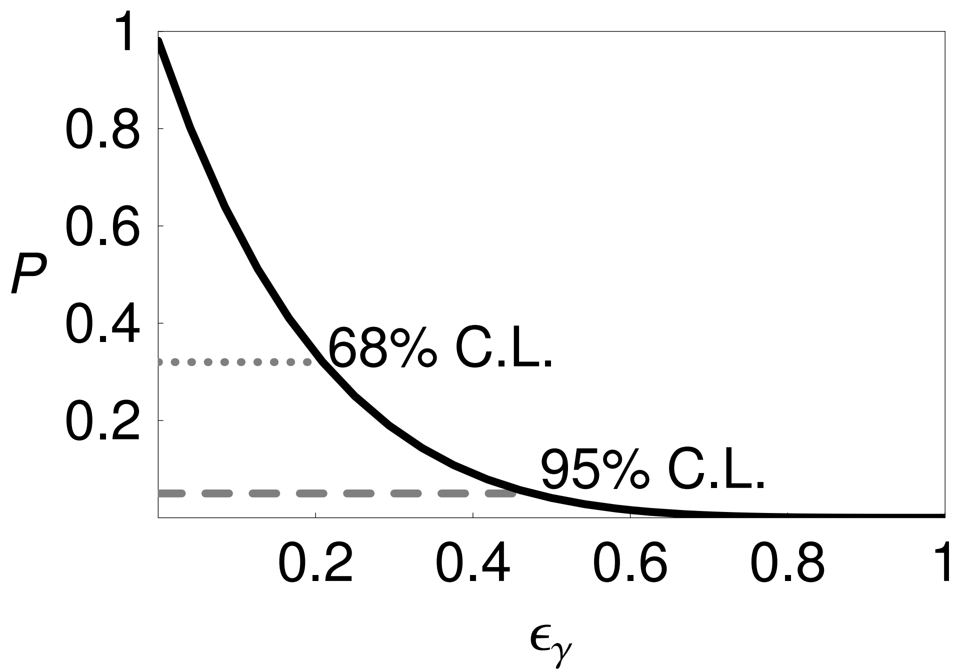
5. Correction because of cuts:

$$\lambda = \frac{m_{\text{lost}}}{m}, \qquad \epsilon_{A,\,\text{true}} = \frac{\epsilon_A}{1 - \lambda + \lambda\epsilon_A}$$

Example

all AGASA events with known muon content
and energies $E_{\rm obs} > 8 \cdot 10^{19}$ eV: N=6

Event	$p_\gamma^{(+)}$	$p_{\overline{\gamma}}^{(+)}$
1	0.000	1.000
2	0.001	0.998
3	0.013	0.921
4	0.003	0.887
5	0.000	0.580
6	0.000	0.565



Result: $\epsilon_{\gamma,\rm true} < 0.50$ at 95% C.L.

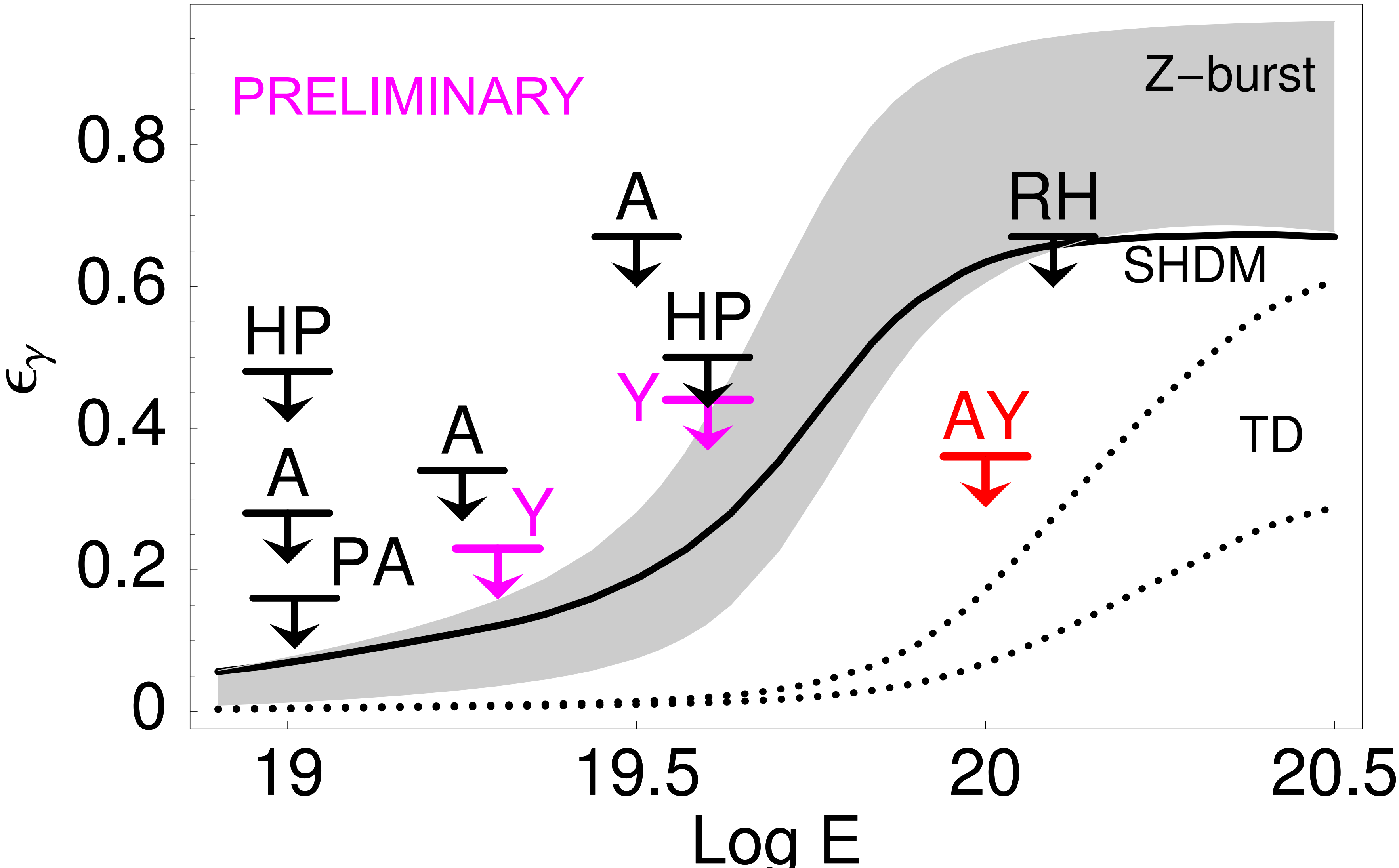
AGASA+Yakutsk, $E > 10^{20}$ eV, muons: $\epsilon_{\gamma,\rm true} < 0.36$

Rubtsov et al, astro-ph/0601449

Low energy Yakutsk data

AY: Rubtsov et al, astro-ph/0601449 $E_Y > 4 \cdot 10^{19}$ eV, $N_Y = 15$

Y: D.G, Rubtsov, Troitsky, Yakutsk Coll., astro-ph/06xxxxx $E_Y > 2 \cdot 10^{19}$ eV, $N_Y = 47$



The most probable chemical composition

1. Input

set of N observed events with energies $\mathcal{E}$,

$p_{A_1}^{(+)j}$, $p_{A_2}^{(+)j}$, generally $p_{A_1}^{(+)j} + p_{A_2}^{(-)j} \neq 1$

Steps

2. Probability $\mathcal{P}(n_1, n_2)$: among N observed events, n_1 initiated by A_1 with $\mathcal{E}$ and n_2 initiated by A_2 with $\mathcal{E}$:

The probability to have i_1-th, . . . , i_{n_1}-th events ($i_1 < \cdots < i_{n_1}$) induced by A_1 with $\mathcal{E}$ and k_1-th, . . . , k_{n_2}-th events ($k_1 < \cdots < k_{n_2}$, $i_j \neq k_l$) induced by A_2 with $\mathcal{E}$:

$$\mathcal{P}\left(\{i_j\},\{k_l\}\right) = \prod_{i_j} p_{A_1}^{(+)i_j} \prod_{k_l} p_{A_2}^{(+)k_l} \prod_{m_n \neq i_j, k_l} \left(1 - p_{A_1}^{(+)m_n} - p_{A_2}^{(+)m_n}\right);,$$

To calculate $\mathcal{P}(n_1, n_2)$ one sums over all subsets $(\{i_j\},\{k_l\})$

$$\mathcal{P}(n_1, n_2) = \sum_{\substack{i_1 < i_2 < \cdots < i_{n_1} \\ k_1 < k_2 < \cdots < k_{n_2}}, i_j \neq k_l} \mathcal{P}\left(\{i_j\},\{k_l\}\right) \,, \qquad 1 \le i_j, k_l, m_n \le N$$

The most probable chemical composition

Let us suppose that $\epsilon_{A_{1,2}}$ are the fractions of $A_{1,2}$

3. Let $\mathcal{P}(\epsilon_{A_1})$ be the probability that the observed results are reproduced for a given set $(\epsilon_{A_1}, \epsilon_{A_2} = 1 - \epsilon_{A_1})$. Hence

$$\mathcal{P}(\epsilon_{A_1}) = \sum_{n_1,n_2}^{n_1+n_2 \leq N} \mathcal{P}(n_1, n_2) \epsilon_{A_1}^{n_1} \left(1 - \epsilon_{A_1}\right)^{n_2}$$

4.a The allowed ϵ_{A_1} at a given confidence level ξ:

$$P(\epsilon_{A_1}) \geq 1 - \xi$$

4.b The most probable composition: maximization of $\mathcal{P}(\epsilon_{A_1})$

5. Correction because of cuts:

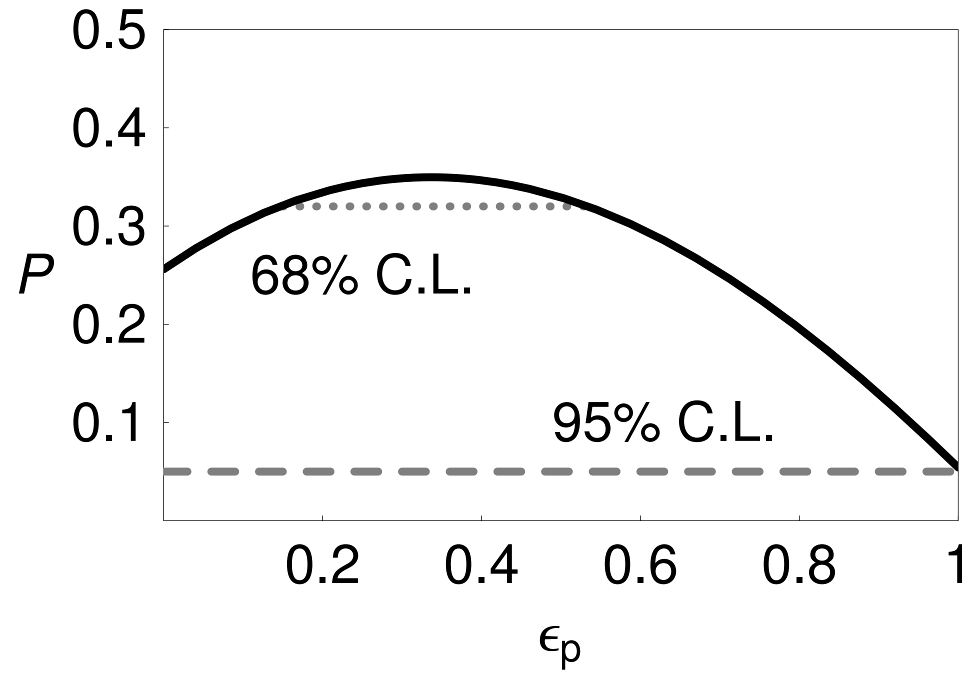
$$\lambda = \frac{m_{\text{lost}}}{m}, \qquad \epsilon_{A_1}^{\text{true}} = \frac{\epsilon_{A_1}(1 - \lambda_{A_2})}{1 - \lambda_{A_1} + \epsilon_{A_1}(\lambda_{A_1} - \lambda_{A_2})}.$$

Example

Two AGASA and two Yakutsk events with known muon content and energies $E_{\rm obs} > 1.5 \cdot 10^{20}$ eV:

N=4 , $A_1 = p$, $A_2 = Fe$

Event	$p_p^{(+)}$	$p_{\rm Fe}^{(+)}$
1	0.254	0.136
2	0.295	0.135
7	0.186	0.814
8	0.413	0.587



Result: most probable $\epsilon_{\rm p} = 0.35$, $\epsilon_{\rm Fe} = 0.65$

$$0.15 < \epsilon_p < 0.54 \quad \text{at } 68\% \text{ C.L.}$$

What else?

With this method one can consider

- Many types of **c**-parameters
- Many types of primaries
- Unknown primary
- Only $\mathcal{E}$ depends on energy systematics