

CKM MATRIX STATUS

J. Rosner – 5th Rencontres du Vietnam – August 6, 2004

Thanks to C.-W. Chiang, M. Gronau, Z. Luo, D. Suprun for enjoyable collaborations

The **Cabibbo–Kobayashi–Maskawa** (CKM) matrix describes the charge-changing weak transitions between quarks with charges $-1/3$ (d, s, b) and $2/3$ (u, c, t):

$$V_{\text{CKM}} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \simeq \begin{bmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 & 1 \end{bmatrix} .$$

Here $\bar{\rho} \equiv \rho(1 - \lambda^2/2)$, $\bar{\eta} \equiv \eta(1 - \lambda^2/2)$. The matrix is unitary: $V^\dagger V = VV^\dagger = 1$.
Parametrization due to Wolfenstein; $\lambda \simeq 0.225$, $A \simeq 0.8$, $\bar{\eta} \simeq 0.36$, $\bar{\rho} \simeq 0.19$.

Many extensive reviews, e.g., Battaglia *et al.* (hep-ph/0304132), Schubert (2003 Lepton-Photon Symposium), Abele *et al.* (hep-ph/0312150), Ali (hep-ph/0312303), Eigen *et al.* (hep-ex/0312062), Ratcliffe [V_{us}] (hep-ph/0402063), Stocchi (hep-ph/0405038), CKMfitter Group (hep-ph/0406184), HFAG 2004, PDG 2004.

Today's review: procedures for determination of elements; open questions.

OBSERVABLES DISCUSSED HERE

Magnitudes:

V_{ud} : Nuclear, neutron, pion β decays

V_{us} : Hyperons, BNL E-865, Fermilab E-832, NA48, KLOE, lattice

V_{cd} : Nonstrange charm decay; neutrino charm production

V_{cs} : strange charm, W decay

V_{cb} : $b \rightarrow c$ inclusive (moments), exclusive (form factors)

V_{ub} : $b \rightarrow u$ inclusive ($b \rightarrow s\gamma$ helps), exclusive ($B \rightarrow \pi\ell\nu, \rho\ell\nu$ form factors)

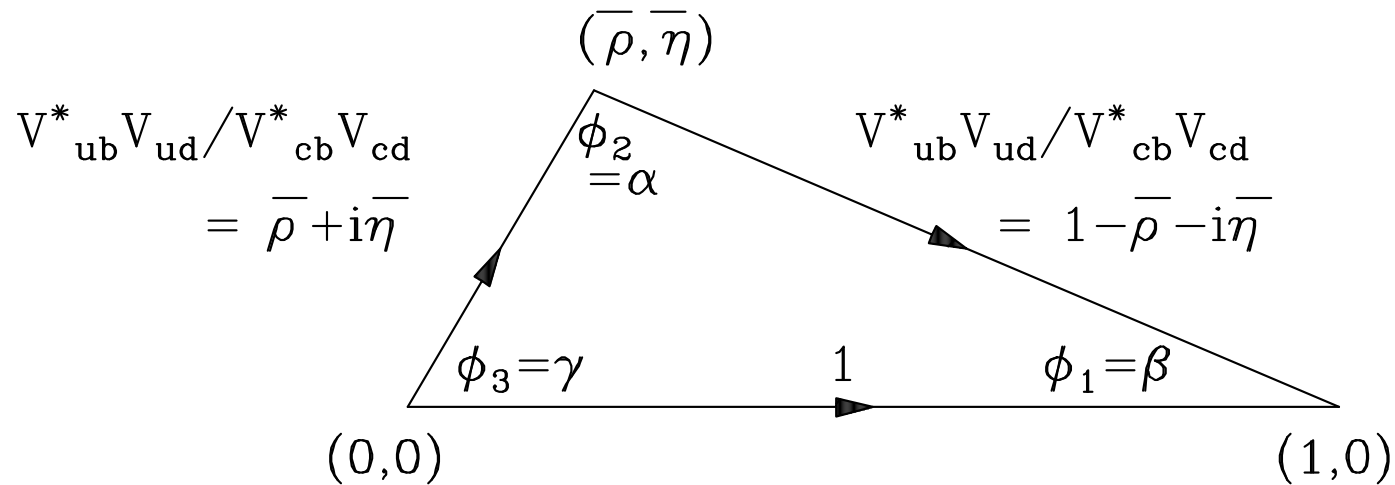
V_{td} : $B-\bar{B}$ mixing (nonstrange and strange); progress on decay constants

V_{ts} : Help from $b \rightarrow s\gamma$, unitarity

V_{tb} : Information from top quark decays, unitarity

Constraints from K_L, K^+ decays: CP violation; $K \rightarrow \pi\nu\bar{\nu}$

UNITARITY TRIANGLE



Phases:

CKMfitter current status ($\pm 2\sigma$):

Processes which (should) give β

$$(23.8_{-3.8}^{+4.5})^\circ$$

α from $B^0 \rightarrow \pi^+ \pi^-$, $\rho^\pm \pi^\mp$, $\rho^+ \rho^-$

$$(94_{-16}^{+24})^\circ$$

γ from global fits to $B \rightarrow PP$, $B \rightarrow VP$ decays

$$(62_{-24}^{+17})^\circ$$

Beyond the 3×3 unitary CKM matrix:

Fourth family

Exclude $100 < m(b') < 199 \text{ GeV}/c^2$

Exotic quarks and leptons

Predicted in E6

V_{ud} FROM β -DECAYS

Czarnecki, Marciano, Sirlin hep-ph/0406324

Nuclear $0^+ \rightarrow 0^+$ transitions (9 measurements) $\rightarrow |V_{ud}| = 0.9740(1)(3)(4)$
(1) expt. [each typically (4)], (3) nuclear theory, (4) radiative corrections

Neutron decay $\Rightarrow |V_{ud}|^2(1 + 3g_A^2)\tau_n = (4908 \pm 4) \text{ s}$

$\tau_n = 885.7(7) \text{ s}$, $g_A = 1.2720(18) \Rightarrow |V_{ud}| = 0.9729(4)(11)(4)$
(4) τ_n , (11) g_A , (4) radiative corrections

Pion beta decay ($\pi^+ \rightarrow \pi^0 e^+ \nu_e$): $|V_{ud}| = 0.9739(39)$; PSI experiment ongoing.

Overall average: $|V_{ud}| = 0.9740(5)$

V_{us} FROM HYPERON DECAYS

Review by N. Cabibbo, BEACH 04 Conference, Chicago, June 28, 2004

New fit by Cabibbo, Swallow, Winston PRL **92**, 251803 (2004):
includes $\Lambda \rightarrow pe^- \bar{\nu}$, $\Sigma^- \rightarrow ne^- \bar{\nu}$, $\Xi^- \rightarrow \Lambda e^- \bar{\nu}$, $\Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}$ (but: g_2 ?)

Combined hyperon value: $|V_{us}| = 0.2250 \pm 0.0027$

V_{us} FROM $K_{\ell 3}$ DECAYS

Brookhaven E-865: $K^+ \rightarrow \pi^0 e^+ \nu \Rightarrow$

$$|V_{us}| = 0.2272 \pm 0.0022_{\text{rate}} \pm 0.0007_{\text{f.f.}} \pm 0.0018_{f_+(0)}$$

A. Sher +, hep-ex/0305042, AIP Conf. Proc. **698**, 381 (2004)

Radiative corrections were an important part of analysis

Fermilab E-832 (KTeV): $K_L \rightarrow \pi \ell \nu \Rightarrow$

$$|V_{us}| = 0.2252 \pm 0.0008_{\text{KTeV}} \pm 0.0021_{\text{ext}}$$

Alexopoulos +, hep-ex/0406001 (see also hep-ex/0406002, hep-ex/0406003) \Rightarrow PRL

Radiative corrections by T. Andre, hep-ph/0406006

$|V_{ud}| = 0.9740 \pm 0.0005$, PDG $|V_{us}| = 0.2196 \pm 0.0026$ agreed poorly with unitarity:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9969 \pm 0.0015$$

while expected value of $|V_{us}|$ from unitarity is

$$|V_{us}| = (1 - |V_{ud}|^2 - |V_{ub}|^2)^{1/2} = 0.2265 \pm 0.0023$$

New results soon: CERN NA48, KLOE at DAΦNE (prelim. $K_S \Rightarrow 0.2194 \pm 0.0030$)

Marciano (hep-ph/0402299) advocates use of lattice $f_K/f_\pi = 1.201(8)(15)$ and $K \rightarrow \mu\nu$, $\pi \rightarrow \mu\nu$ to extract $|V_{us}| = 0.2236(5)(30)$ (non-lattice) (lattice)

V_{cd} FROM CHARM

Decays (also V_{cs}):

CLEO II.V: $D^0 \rightarrow \pi^- \ell^+ \nu$ and $D^0 \rightarrow K^- \ell^+ \nu \Rightarrow$

$$|f_+^\pi(0)|^2 |V_{cd}|^2 / |f_+^K(0)|^2 |V_{cs}|^2 = 0.038_{-0.007}^{+0.006+0.005}$$

Fermilab-MILC Collaboration (Okamoto 7/23/04): $n_f = 3$ lattice QCD \Rightarrow

$$f_+^{D \rightarrow \pi}(0) = 0.64(3)(5) , \quad f_+^{D \rightarrow K}(0) = 0.73(3)(6) , \quad \text{errors (stat)(syst) ;}$$

$$|V_{cd}| = 0.239(10)(19)(20) , \quad |V_{cs}| = 0.969(39)(78)(24) .$$

Production by neutrinos (measure dileptons):

$$\overline{\mathcal{B}}(c \rightarrow \ell + X) |V_{cd}|^2 = (4.63 \pm 0.34) \times 10^{-3} ; \quad \overline{\mathcal{B}}(c \rightarrow \ell + X) = (9.23 \pm 0.73)\%$$

$$\Rightarrow |V_{cd}| = 0.224 \pm 0.012$$

CLEO-c: $\mathcal{B}(D^0 \rightarrow \pi^- e^+ \nu) = (3.73 \pm 0.10 \pm 0.20)\%$ with $\sim 57 \text{ pb}^{-1}$

Higher-luminosity running in $e^+e^- \rightarrow \psi''(3770) \rightarrow D\bar{D}$ to begin in September.

Hoping for 3 fb^{-1} . Lattice errors will be limiting factor in extracting $|V_{cd}|$ to $\mathcal{O}(1\%)$.

V_{cs} FROM W DECAYS

Charm-tagged W decays at LEP II (ALEPH, DELPHI):

$$|V_{cs}| = 0.97 \pm 0.09 \pm 0.07$$

Leptonic branching ratio $\mathcal{B}(W \rightarrow \ell\nu)$, no unusual W decays:

$$\frac{1}{\mathcal{B}(W \rightarrow \ell\nu)} = 3 \left(1 + \left[1 + \frac{\alpha_s(M_W)}{\pi} \right] \sum_{\substack{i=u,c \\ j=d,s,b}} |V_{ij}|^2 \right)$$

Implies $\sum |V_{ij}|^2 = 2.039 \pm 0.025$ and hence $|V_{cs}| = 0.996 \pm 0.013$

when contributions of other CKM elements are subtracted

Cabibbo-favored ($c \rightarrow s$) decays of charmed particles at CLEO-c:

Will provide $|V_{cs}|$ at $\mathcal{O}(1\%)$ accuracy if \mathcal{L} goal achieved and lattice cooperates

Can't see violation of two-family unitarity:

$$|V_{ud}|^2 + |V_{us}|^2 = |V_{cd}|^2 + |V_{cs}|^2 = |V_{ud}|^2 + |V_{cd}|^2 = |V_{us}|^2 + |V_{cs}|^2 = 1 .$$

Reflects on the very hierarchical structure of the CKM matrix.

V_{cb} : INCLUSIVE $b \rightarrow c$ DECAYS

At the quark level, $b \rightarrow c$ semileptonic decay rate is simple (neglecting m_ℓ):

$$\Gamma(b \rightarrow c\bar{\nu}_\ell\ell^-) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 f\left(\frac{m_c^2}{m_b^2}\right),$$

where $f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$

Real hadrons in initial and final states: perturbative QCD; expansions in inverse heavy quark mass (“HQE”); lepton energy, hadron mass, $b \rightarrow s\gamma$ moments \Rightarrow

$$|V_{cb}| = 0.0421 \pm 0.0013 \quad (\text{Schubert's 2003 average})$$

but quoted by CKMfitter group as $0.0420 \pm 0.0006_{\text{stat}} \pm 0.0008_{\text{theo}}$.

More recently (e.g.):

$$\text{BaBar: } |V_{cb}| = (41.4 \pm 0.4_{\text{stat}} \pm 0.4_{\text{HQE}} \pm 0.6_{\text{theo}}) \times 10^{-3}$$

$$\text{CLEO: } |V_{cb}| = (42.4 \pm 0.8_{\text{stat+HQE}} \pm (> 0.8)_{\text{theo}}) \times 10^{-3}$$

These errors are smaller than those in exclusive analyses; view with caution

CKMfitter's inclusive-exclusive average: $|V_{cb}| = (41.1^{+1.4}_{-0.6}) \times 10^{-3}$ ($\sim \sqrt{\text{Schubert}}$)

V_{cb} : EXCLUSIVE $b \rightarrow c$ DECAYS

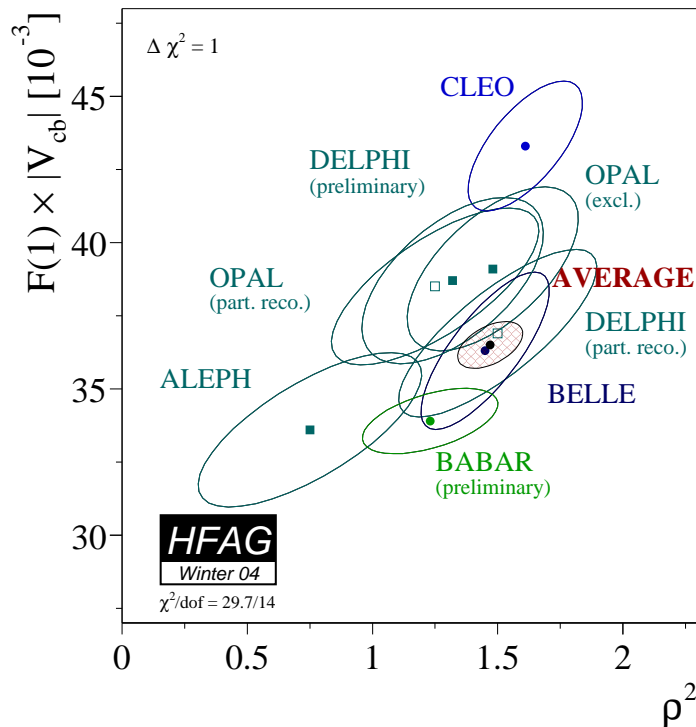
Best determination comes from $B \rightarrow D^* \ell \bar{\nu}_\ell$:

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3} |V_{cb}|^2 (m_B - m_{D^*}^*)^2 m_{D^*}^3 \sqrt{w^2 - 1} \mathcal{G}(w) |\mathcal{F}(w)|^2$$

One Isgur-Wise form factor $\mathcal{F}(w) \simeq \mathcal{F}(1)[1 + \rho^2(w - 1) + \dots]$

Isgur-Wise variable $w = v_B \cdot v_{D^*}$; phase space $\mathcal{G}(w)$ with $\mathcal{G}(1) = 1$:

$\mathcal{F}(1) = \eta_{QCD}[1 + \mathcal{O}(1/m_b^2)] = 0.913_{-0.035}^{+0.030}$ (Lattice, HQET).



Latest $\mathcal{F}(1)|V_{cb}|^2 = (36.5 \pm 0.8) \times 10^{-3}$

$\rho^2 = 1.47 \pm 0.13$: form factor slope parameter

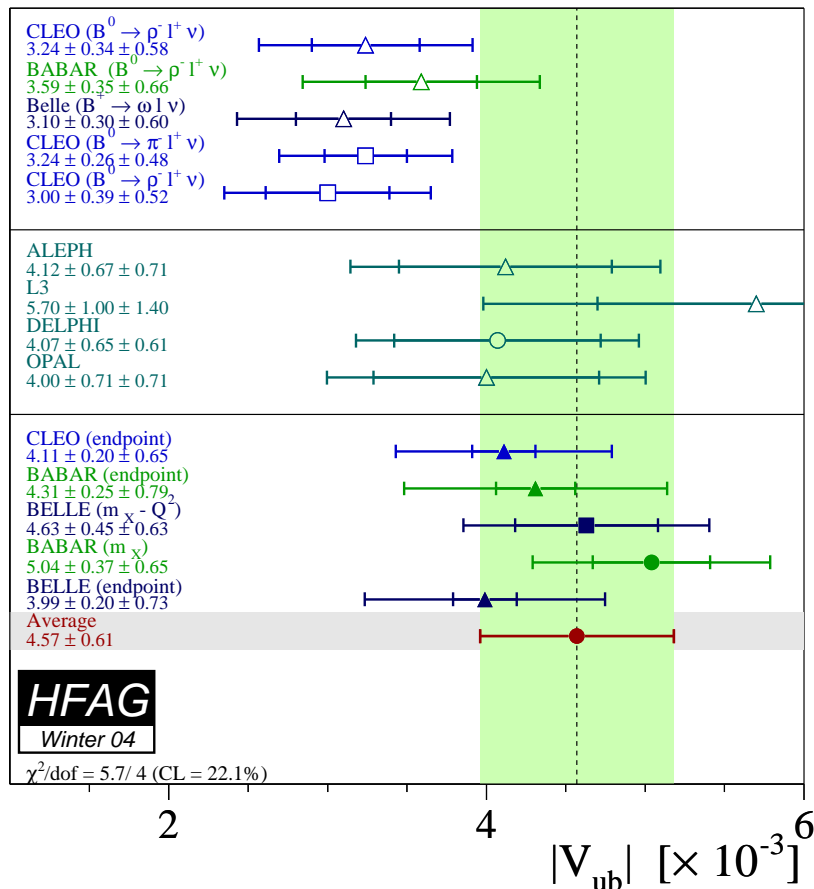
$|V_{cb}| = (40.1 \pm 0.9_{\text{exp}} \pm 1.8_{\text{th}}) \times 10^{-3}$

Form fact. shape fits $B^0 \rightarrow D^{(*)-} \pi^+$, $D^{(*)-} D_s^{(*)+}$ rates via factorization (PR D **64**, 094001)

Consistent with inclusive $|V_{cb}|$ but larger errors

V_{ub} : INCLUSIVE $b \rightarrow u$ DECAYS

Semileptonic decay $b \rightarrow u\bar{\nu}_\ell\ell^-$ again the source of information, but $\Gamma(b \rightarrow u\bar{\nu}_\ell\ell^-)$ is only about 2% of $\Gamma(b \rightarrow c\bar{\nu}_\ell\ell^-)$. Several strategies have been used:



Leptons beyond $b \rightarrow c\bar{\nu}_\ell\ell^-$ end point

Reconstruct hadronic mass $M_X < M_D$

Cut on $q^2 = m_{\ell\nu}^2$

Light-cone-inspired kinematic cut

As in $b \rightarrow c\bar{\nu}_\ell\ell^-$, $b \rightarrow s\gamma$ photon spectrum pins down hadronic uncertainties

Bottom 5 points:

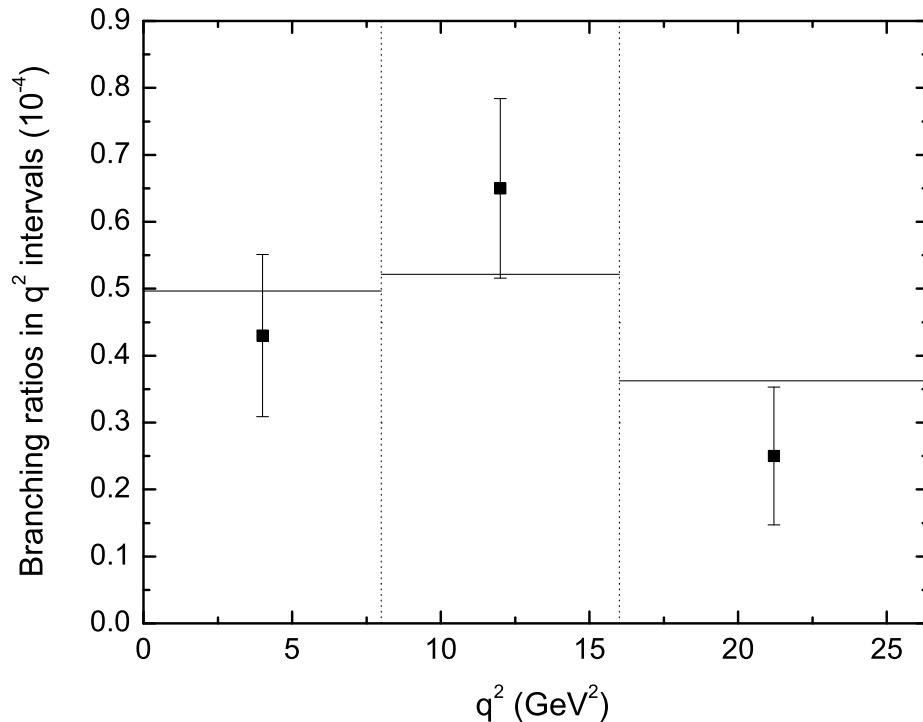
$$|V_{ub}| = (4.57 \pm 0.61) \times 10^{-3}$$

Notably higher than value from exclusive $b \rightarrow u$ decays (e.g., $B \rightarrow \pi\bar{\nu}_\ell\ell$):

Top 5 plotted points (my average) $\Rightarrow |V_{ub}| = (3.23 \pm 0.62) \times 10^{-3}$

V_{ub} : EXCLUSIVE $b \rightarrow u$ DECAYS

CLEO $B \rightarrow \pi \bar{\nu}_\ell l$ spectrum:



In exclusive average I assigned overall ± 0.60 systematic error

Combining with inclusive value, including scale $S = 1.54 \Rightarrow$

$$|V_{ub}| = (3.91 \pm 0.66) \times 10^{-3}$$

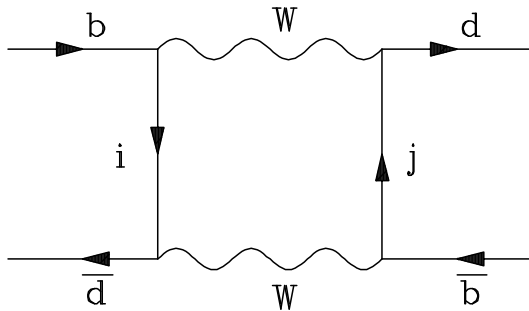
Discrepancy between inclusive, exclusive values merits caution (quark-hadron duality valid?)

Fit to CLEO $\int dq^2 \frac{d\mathcal{B}}{dq^2}(B^0 \rightarrow \pi^- l^+ \nu_l)$ spectrum helped relate high- q^2 lattice QCD form factor predictions to q^2 behavior needed for testing factorization: Z. Luo + JLR, PR D **68**, 074010 (2003)

V_{td} FROM $B-\bar{B}$ MIXING

Loop diagram allows $b\bar{d} \leftrightarrow d\bar{b}$ transitions (2nd order weak)

Quarks $i, j = u, c, t$ in loop; t dominates \Rightarrow information on $|V_{td}|$



Crossed diagram contributes $\times 1/3$

f_B^2 (B meson decay constant) governs matrix element of $b\bar{d} \leftrightarrow d\bar{b}$ operator

Parameter $B_B = 1$ if W exchange diagrams dominate

$f_B\sqrt{B_B} = (228 \pm 30 \pm 10)$ MeV and $\Delta m_d \simeq 0.5 \text{ ps}^{-1} \Rightarrow 95\%$ c.l. range

$$|V_{td}| = (8.26_{-1.79}^{+1.23}) \times 10^{-3} \Leftrightarrow |1 - \bar{\rho} - i\bar{\eta}| = 0.89_{-0.20}^{+0.12}$$

CKMfitter 95% c.l. range of $|V_{ub}| = (3.87_{-0.61}^{+0.73}) \times 10^{-3} \Rightarrow |\bar{\rho} + i\bar{\eta}| = 0.40_{-0.06}^{+0.08}$

Forthcoming information from $\mathcal{B}(B \rightarrow \rho\gamma)/\mathcal{B}(B \rightarrow K^*\gamma)$;
expect $\mathcal{B}(B^0 \rightarrow \rho^0\gamma) = (0.64 \pm 0.23) \times 10^{-6}$ (cleaner than $\rho^+\gamma$)

CLEO $f_D = (204 \pm 42 \pm 17)$ MeV: Accuracy already approaching that of lattice

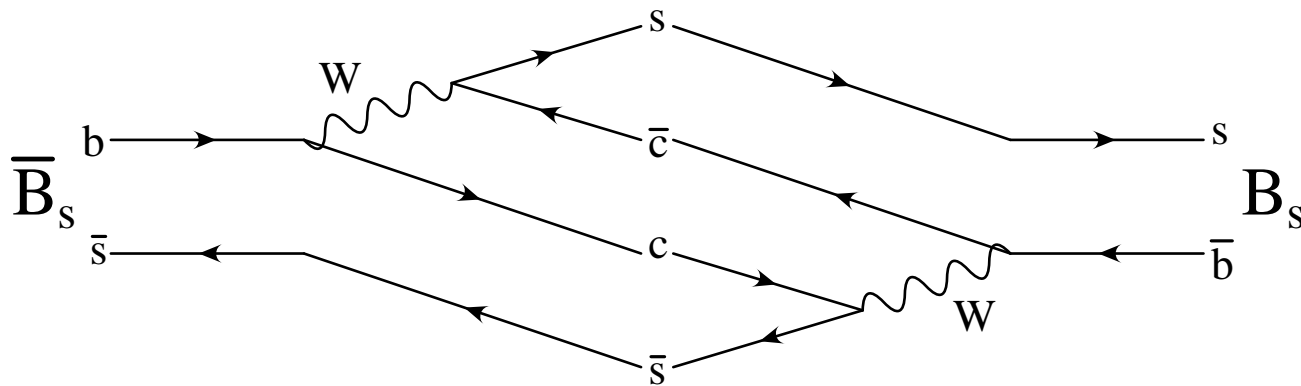
$B_s - \bar{B}_s$ MIXING

Asymmetric error in $|V_{td}|$: restricted on positive side by $B_s - \bar{B}_s$ mixing

Same diagram as before but substitute $d \rightarrow s$ and assume $V_{ts} \simeq -V_{cb}$

Lower limit $\Delta m_s > 14.5 \text{ ps}^{-1}$ and $\xi \equiv f_{B_s} \sqrt{B_{B_s}} / f_B \sqrt{B_B} = 1.21 \pm 0.06 \Rightarrow$
lower limit on V_{ts}/V_{td} and hence upper limit on V_{td}

Global CKMfitter fit $\Rightarrow 2\sigma$ prediction $\Delta m_s = 17.8_{-2.7}^{+15.2} \text{ ps}^{-1}$



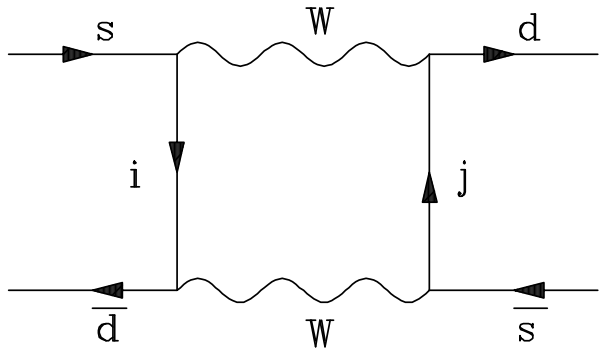
Mixing via shared intermediate states $\Rightarrow \Delta\Gamma_s \simeq -\Delta m_s/200$ ($\sim m_b^2/m_t^2$) or $\Delta\Gamma/\bar{\Gamma} \simeq 0.18(f_{B_s}/200 \text{ MeV})^2$

CDF has recently reported a value of $0.65_{-0.23}^{+0.25} \pm 0.01$ (C. Gay, this conference)

K_L CP VIOLATION

CP-violating $K^0-\bar{K}^0$ mixing dominated by top quarks in loop; charm also

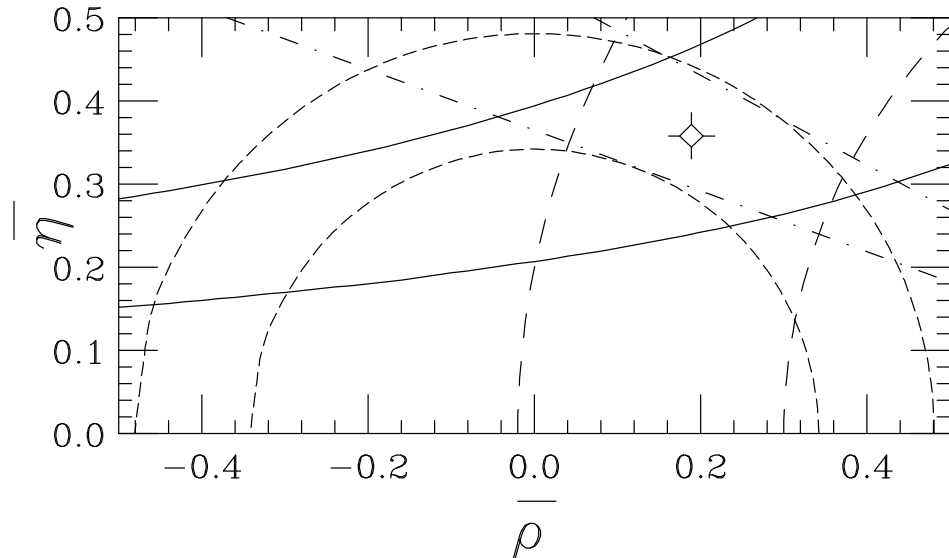
Parameter ϵ_K governing CP violation mostly $\sim \text{Im}(V_{td}^2)$ and hence measures $\bar{\eta}(1-\bar{\rho})$



Convenient expressions in Ali, hep-ph/0312303, imply $(\bar{\rho}, \bar{\eta})$ lie between boundaries

$$\bar{\eta}(1.38 - \bar{\rho}) = 0.28, \quad \bar{\eta}(1.26 - \bar{\rho}) = 0.50$$

Constraints on $(\bar{\rho}, \bar{\eta})$ from $|V_{ub}|$ (dotted circles), $|V_{td}|$ (dashed circles), ϵ_K (solid hyperbolae, 1σ), $\sin 2\beta$ (dotdash) select region around $(0.19, 0.36)$ (plotted point)



Point: $(0.19_{-0.07}^{+0.09}, 0.36_{-0.04}^{+0.05})$

Note consistency

Main uncertainty: $|1 - \rho - i\eta|$

Direct CPV (ϵ'/ϵ) seen but no constraint (hadronic uncertainty)

$K \rightarrow \pi \nu \bar{\nu}$ CONSTRAINTS

Higher-order weak diagrams govern weak quark transition $s \rightarrow d \nu \bar{\nu}$. Top quark in loop dominates but also charm contribution \Rightarrow result measures $|1.3 - \bar{\rho} - i\bar{\eta}|$

$K^+ \rightarrow \pi^+ \nu \bar{\nu}$: Brookhaven experiment E-787, E-949 (hep-ex/0403036) sees 3 events

Branching ratio $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (1.47_{-0.82}^{+1.30}) \times 10^{-10}$ vs. Standard Model prediction $(0.78 \pm 0.12) \times 10^{-10}$ (Buras et al., hep-ph/0405132).

Still consistent with large region in $(\bar{\rho}, \bar{\eta})$.

Proposals for Fermilab, JPARC seek a sample of 100 events $\Rightarrow |1.3 - \bar{\rho} - i\bar{\eta}|$ to 5%

$K_L \rightarrow \pi^0 \nu \bar{\nu}$ is purely CP-violating and measures $\bar{\eta}^2$: expect $\mathcal{B} = (3.0 \pm 0.6) \times 10^{-11}$ (hep-ph/0405132)

KEK PS E-391 has taken data whose single-event sensitivity should be an order of magnitude above Standard Model value; tremendous progress in past few years

Eventual experiment at JPARC or Brookhaven: goal of seeing Standard Model rate

V_{ts} AND V_{tb}

Lower limit on $|V_{tb}^* V_{ts}|$ from $B_s - \bar{B}_s$ mixing, theoretical constraint on $\xi \equiv f_{B_s} \sqrt{B_{B_s}} / f_B \sqrt{B_B} = 1.2 \pm 0.1$, $B_d - \bar{B}_d$ mixing

Ali (hep-ph/0312303) concludes $|V_{tb}^* V_{ts}| > 0.034$

Upper limit on $|V_{tb}^* V_{ts}|$ from top quark contribution to $b \rightarrow s \gamma$

Ali + Misiak (hep-ph/0304132) $\Rightarrow V_{tb}^* V_{ts} = -0.047 \pm 0.008$ (expected sign)

Top quark decays $t \rightarrow b \ell^+ \nu_\ell$ dominate over those with no b (CDF):

$$\frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2} = 0.94_{-0.24}^{+0.31} .$$

CDF Run II (108 pb⁻¹): $0.54_{-0.39}^{+0.49}$

Assumption that CKM matrix is 3×3 is crucial to interpretation of this result as a useful bound on $|V_{tb}|^2$; see PDG review for discussion of almost non-existent lower bounds on 3rd row if there are more than 3 families.

PROCESSES GIVING β

CP asymmetry $A(t) = -C \cos(\Delta mt) + S \sin(\Delta mt)$

Penguin-dominated; expect $C = 0$, $S = \sin(2\beta) = 0.74 \pm 0.05$ (points plotted below)

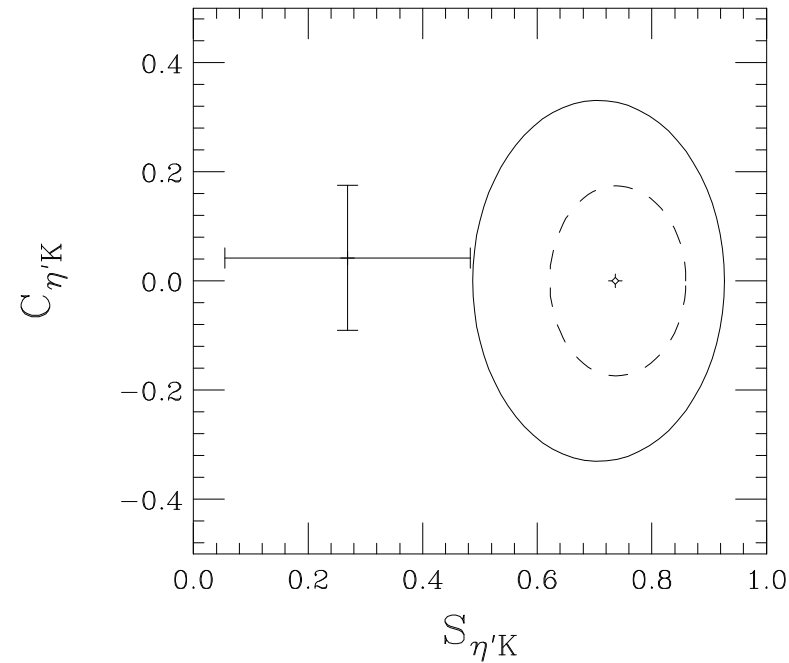
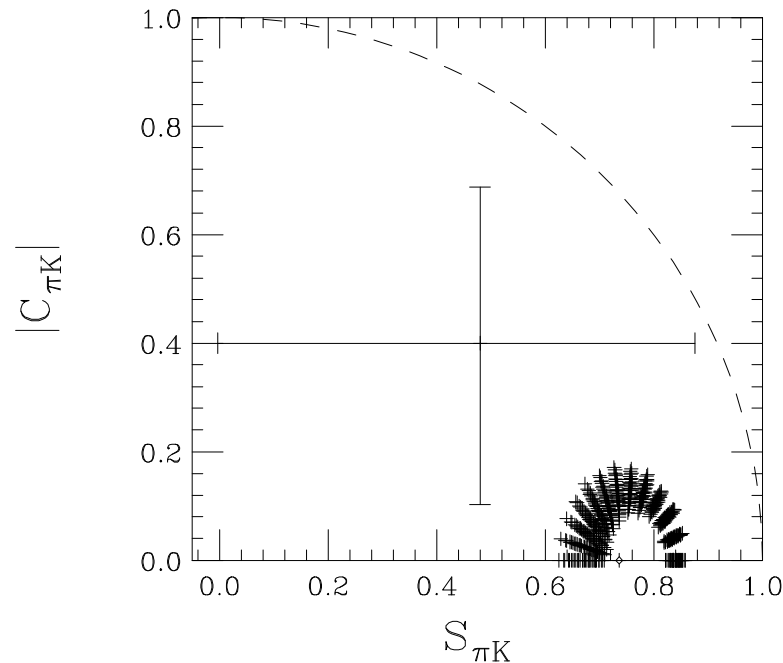
Contributions from other amplitudes using new measurements and flavor SU(3)

M. Gronau + PL B **579**, 331 (2004)

PL B **596**, 107 (2004)

$B^0 \rightarrow \pi^0 K_S$

$B^0 \rightarrow \eta' K_S$



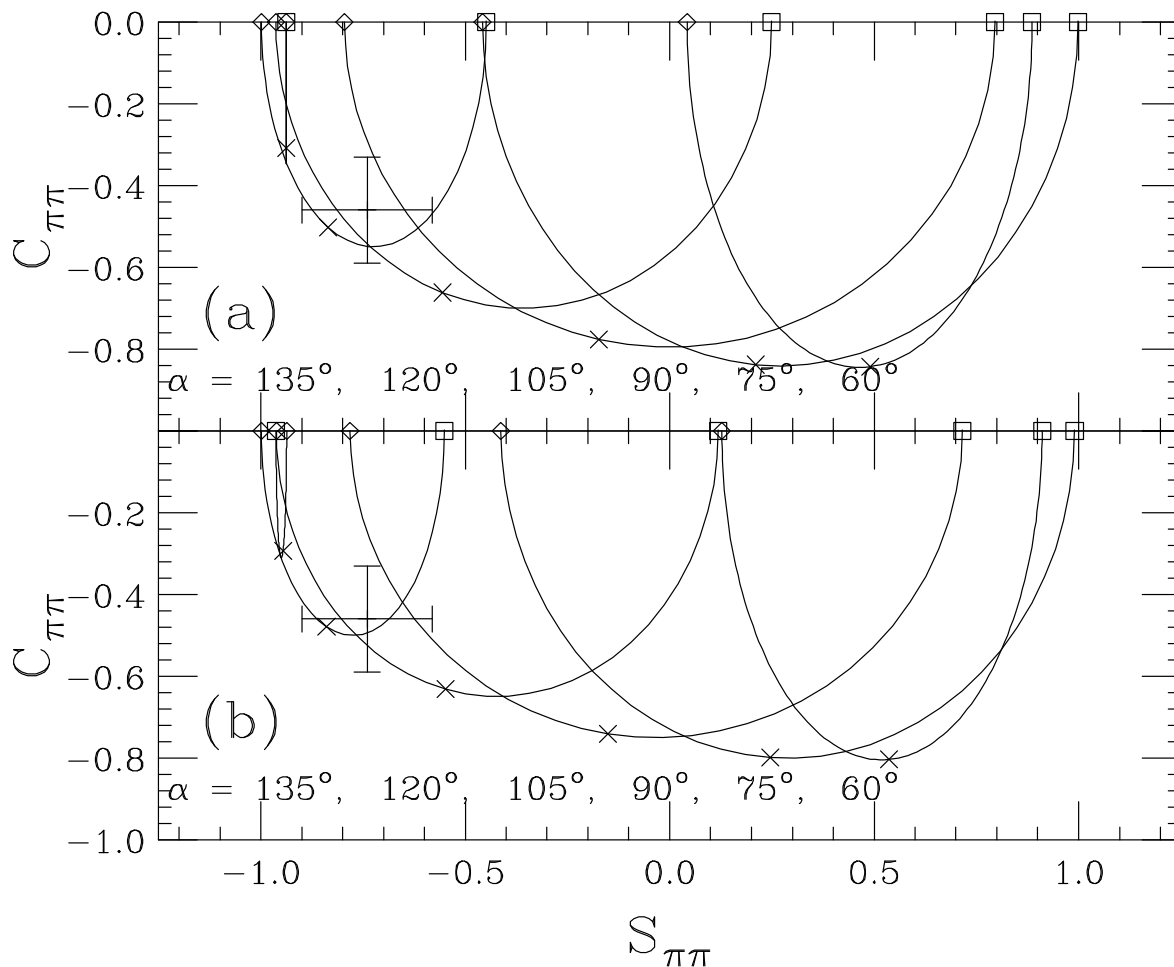
$B^0 \rightarrow \pi^0 K_S$: Hatched ellipse = allowed region. $B^0 \rightarrow \eta' K_S$, solid curve: bounds based on flavor SU(3) and new measurements of other processes; dashed curve: neglecting processes involving spectator quark.

$B^0 \rightarrow \pi^+\pi^-$, ... AND α

$\pi^+\pi^-$: M. Gronau + JLR, Phys. Lett. B **595**, 339 (2004)

$$\frac{\Gamma(\bar{B}^0(t) \rightarrow \pi^+\pi^-) - \Gamma(B^0(t) \rightarrow \pi^+\pi^-)}{\Gamma(\bar{B}^0(t) \rightarrow \pi^+\pi^-) + \Gamma(B^0(t) \rightarrow \pi^+\pi^-)} = -C_{\pi\pi} \cos(\Delta mt) + S_{\pi\pi} \sin(\Delta mt)$$

Penguin pollution ($\sim 70\%$) from (a) $B^+ \rightarrow K^0\pi^+$ or (b) $B^0 \rightarrow K^+\pi^-$



Contours: strong phase
 $0^\circ > \delta > -180^\circ$

Best bounds from (b):
 $\alpha = (103 \pm 17)^\circ$

Gronau + Zupan,
 $B^0 \rightarrow \rho^\pm\pi^\mp$,
hep-ph/0407002:
 $\alpha = (95 \pm 16)^\circ$

BaBar: $B^0 \rightarrow \rho^+\rho^-$
 $\alpha = (96 \pm 10 \pm 4 \pm 13)^\circ$

$B \rightarrow PP, VP$ AND γ

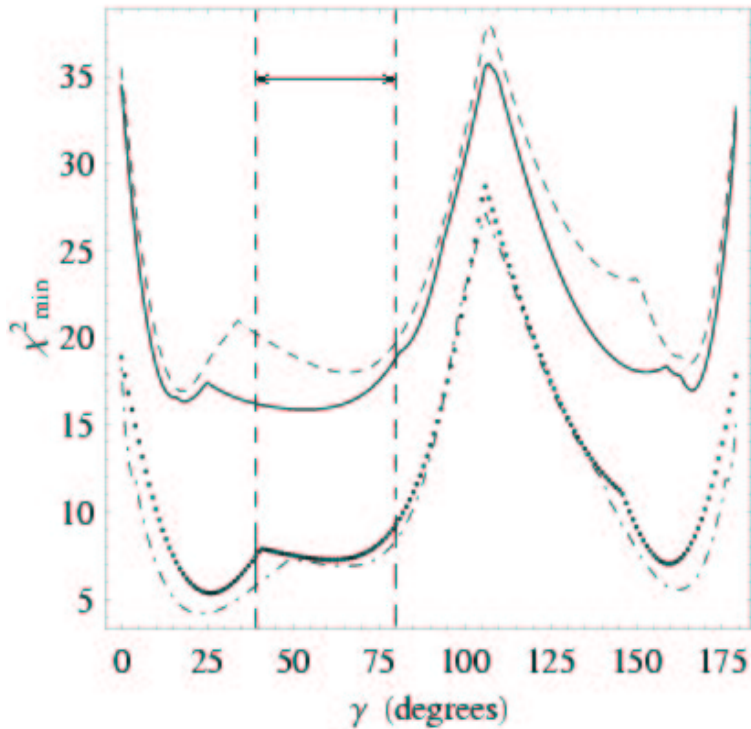
P = light pseudoscalar meson, V = light vector meson (u, d, s only)

Fits to rates and CP asymmetries in flavor SU(3) (T, P, C, \dots amplitudes)

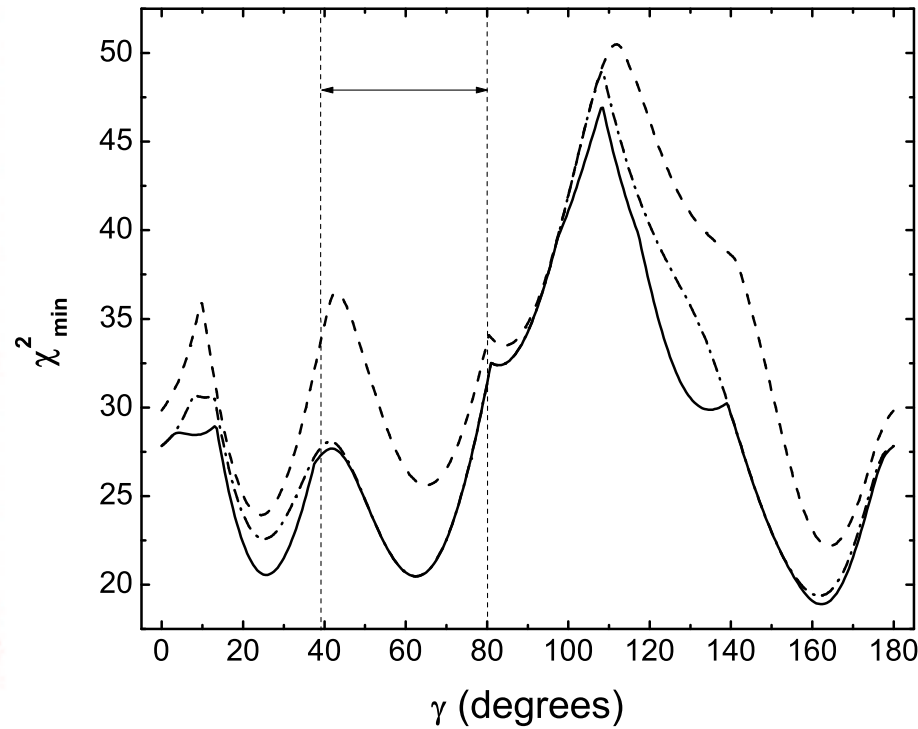
C.-W. Chiang *al.*, PP : hep-ph/0404073 (PR D); VP : PR D **69**, 035009 (2004)

$B \rightarrow PP$: 26 observables

$B \rightarrow VP$: 34 observables



Solid: $\chi^2_{\min}/\text{d.f.} = 16.0/13$



Solid: $\chi^2_{\min}/\text{d.f.} = 20.5/22$

VP alone: $\gamma = (63 \pm 6)^\circ$; combined (D. Suprun's Ph. D. thesis): $\gamma \simeq 62^\circ$

BEYOND THE 3×3 CKM MATRIX

A fourth family?

Fourth neutrino, if it exists, must be heavy: only 3 light ν in Z decay (e.g., $N_\nu = 2.98 \pm 0.064$ from $e^+e^- \rightarrow \gamma + X$ above the Z , L3 at LEP)

Direct search for b' heavier than Z (look for $b' \rightarrow bZ$) by CDF at Tevatron, PRL **84**, 835 (2000). Exclude $100 < m(b') < 199 \text{ GeV}/c^2$

Outside the familiar pattern:

Quark–lepton families: 16-dimensional multiplets of grand unified group $SO(10)$.

16 of $SO(10)$ is **1** + **5*** + **10** of $SU(5)$; **27** of E_6 is **16** + **10** + **1** of $SO(10)$.

10 of $SO(10)$: isosinglet quarks “ h, \bar{h} ” (charge $Q = \pm 1/3$), isodoublet leptons.

The $SO(10)$ singlets are candidates for sterile neutrinos, one for each family.

Exotic h quarks can mix with b and push its mass down with respect to t .

$h\bar{h}$ production signatures: T. Andre + JLR, PR D **69**, 034001 (2004):

Decays to $Z + b$, $W + t$, possibly Higgs + b ; reach 270–320 GeV/c^2 at Tevatron

SUMMARY

No problem with CKM unitarity relation $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

Improved V_{cd} and V_{cs} still consistent with $V_{cd} = -V_{us}$ and $V_{cs} = V_{ud}$

Errors on $V_{cb} \sim 41\text{--}42 \times 10^{-3}$ from inclusive analyses are being reduced; lattice QCD will help exclusive analyses catch up

Lower V_{ub} in exclusive ($\simeq 3.2 \times 10^{-3}$) than inclusive ($\simeq 4.6 \times 10^{-3}$) $b \rightarrow u$ decays. Need better understanding of form factors (exclusive), quark-hadron duality (inclusive); $\Rightarrow \sim 3.9 \times 10^{-3}$

Errors in $|V_{td}| \simeq (8.3_{-1.8}^{+1.2}) \times 10^{-3}$ mainly lattice errors in $f_B\sqrt{B_B}$

Major improvement expected from detection of $B_s\text{--}\bar{B}_s$ mixing

SU(3) fit $\Rightarrow \gamma = (63 \pm 6)^\circ$ needs validation, e.g., via $b \rightarrow d$ penguins

No evidence against $V_{ts} \simeq -V_{cb}$, $V_{tb} \simeq 1$

It's time for a theory of quark masses and CKM elements!