MSGUT : from Futility to Precision

Charanjit S. Aulakh

Dept. of Physics, Panjab University
Chandigarh, INDIA

CSA and A. Girdhar: **HEP-PH/0204097; 0405074**

- MSGUT

  *Virtues

  *Structure
- Spectrum (a la PS)
- Threshold Corrections to $Sin^2\theta_W, M_X$
- New $\Delta B \neq 0$ Channels
- Fermion Mass Formulae
- Conclusions

SO(10) MSGUT : Virtues

- SUSY SO(10) : $\mathbf{210}, \overline{\mathbf{126}}, \mathbf{126}, \mathbf{10}$
  CSA,Mohapatra; Clark,Kuo,Nakagawa (1982)
  * $\oplus 120(NMSGUT)$, Oshimo, Bertolini et al

- $Q_L, Q_L^c \oplus L_L, l_L^c \oplus \nu_L^c \in 16_F$

- MSLRM :$SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \in SO(10)$
  * $R_p \sim M_p = (-)^{3(B-L)} \surd \Rightarrow$ stable LSP ($< \Delta(B-L = \pm 2) > \in \overline{126}$)
  1996 - 2001: CSA,Benakli,Senjanovic,Melfo,Rasin,Bajc....

- $< M_{B-L} > \sim M_X \sim 10^{14} - 10^{16} GeV$

- *SEESAW I : $\nu_L^c, M_{B-L} \sim M_X \Rightarrow M_\nu \sim \frac{m_D^2}{M_{B-L}} \sim 10^{-1} eV$
  *SEESAW II : $< \Delta[1,3,\pm 2] > \sim \frac{v_W^2}{M_\Delta} \Rightarrow M_\nu \sim \frac{f v_W^2}{M_\Delta}$

- Babu, Mohapatra (1992) : 10+126 $\Rightarrow M_\nu, m_u, m_d, m_l, \theta_q, \theta_l$...
  PREDICTIVE (& falsified!)
  *Bajc,Senjanovic, Vissani (2002) : Type II $\Rightarrow$ $\theta_\nu >> \theta_q \surd\surd$
  *Bertolini et al (2004) $\oplus 120$: still predictive ; GUT SSB undisturbed ; falsifiable ?

- **MINIMAL HARD PARAMETERS**
  * MSSM : $\mathbf{26} = 6 m_{u,d} + 6 m_{l,\nu} + 6\theta_{Q,L} + 1\delta_q + 3\delta_L + 3 g_i + \mu$
  *MSGUT : $9 W_{AM} + 16 W_{FM} + 1 g = \mathbf{26}$

* $SU(5) \times R_p$ Competitors :
a) NR : **33**
b) Dirac M_ν: **42**
c) Ltriplets$(15+\overline{15})$: **39** .

- **ASYMPTOTIC STRENGTH !?** :

$$\beta(g) = \frac{109}{16\pi^2} g^3 + O(g^5)....$$

* ASGUT SCENARIO CSA hep-ph/0207150,0210337.

***Dynamical Symmetry Breaking** :
G_{123} Singlet Condensate

$$< \Phi > \sim (calculable\#)\Lambda_U$$

$g(M_S), M_X, M' \Rightarrow \Lambda_U$

* Dual Unification :
Dual GUT monopoles $\sim Q, L$

$$\tilde{g} \sim 1/g$$

$$\Lambda_U \sim \tilde{M}_X$$

$$\tilde{\Lambda}_U \sim M_X$$

Dual Monopole Cores ∼ “Quark Hearts” !

The **SO(10) MSGUT : Structure**

- AMHiggs: $\Phi(210), \overline{126}, 126$
- FMHiggs : $\overline{\Sigma}(\overline{126}), \Sigma(126), H(10)$
- $W_{AM} = M_H H^2 + m\Phi^2 + \lambda\Phi^3 + M\Sigma\overline{\Sigma} + \eta\Phi\Sigma\overline{\Sigma} + H\Phi(\gamma\Sigma + \overline{\gamma}\overline{\Sigma})$ (**9** parameters)
- $W_{FM} = h'_{AB}\psi_A\psi_B H + f'_{AB}\psi_A\psi_B\overline{\Sigma}$ (**16** parameters).
- GUT scale vevs : $SO(10) \rightarrow MSSM$ **AM,CKN82**:

$$\langle(1,1,1)\rangle_{210} = p \quad ; \quad \langle(15,1,1)\rangle_{210} = \quad ; \quad \langle(15,1,3)\rangle_{210} = \omega$$
$$\langle(10,1,3)\rangle_{\overline{126}} = \bar{\sigma}; \qquad \langle(\overline{10},1,3)\rangle_{126} = \sigma$$

- Superpot vev:

$$W = m(p^2+3a^2+6\omega^2)+2\lambda(a^3+3p\omega^2+6a\omega^2)+(M+\eta(p+3a-6\omega))\sigma\overline{\sigma}$$

- EM : $\quad < F >= 0; \qquad D = 0$

$$\begin{aligned}
2mp + 6\lambda\omega^2 + \eta\sigma\bar{\sigma} &= 0\\
2ma + 2\lambda(a^2 + 2\omega^2) + \eta\sigma\bar{\sigma} &= 0\\
2m\omega + 2\omega\lambda(p + 2a) - \eta\sigma\bar{\sigma} &= 0\\
(M + \eta(p + 3a - 6\omega))\sigma &= 0\\
\sigma = \bar{\sigma}
\end{aligned}$$

- Solutions: *SSB completely analyzable !!* **ABMSV 03**

  Non-Degenerate : Cubic in $x = -\lambda\omega/m$:

$$8x^3 - 15x^2 + 14x - 3 = -\xi(1-x)^2$$

Single control parameter !: $\xi = \frac{\lambda M}{\eta m}$

(Units of $\tilde{m} = m/\lambda \Rightarrow \tilde{}$)

$$\tilde{a} = \frac{(x^2+2x-1)}{(1-x)} \quad ; \quad \tilde{p} = \frac{x(5x^2-1)}{(1-x)^2}$$

$$\tilde{\sigma}\tilde{\bar{\sigma}} = \frac{2\lambda}{\eta}\frac{x(1-3x)(1+x^2)}{(1-x)^2}$$

- **Degenerate cases**

  *

$$SU(5) : p = a = -\omega, \sigma \neq 0;$$

*

$$SU(5) : p = a = \omega, \sigma = 0;$$

*

$$(SU(5) \times U(1)_X)_{flipped} : p = a = \omega, \quad \sigma = 0$$

*

$$G_{LR} : p = -3a/2 = \pm 3i\omega = 3m/\lambda, \quad \sigma = 0$$

AM Chiral masses via PS

- SO(10) a la $G_{PS} = SU(4) \times SU(2)_L \times SU(2)_R$
- $$10(i) = (a = 1..6, \tilde{\alpha} = 7...10) \sim ([\mu\nu], \alpha\dot{\alpha})$$
$$16 = ((\mu, \alpha), (\bar{\mu}, \dot{\alpha}))$$
- $$V_i W_i \quad = \quad \frac{1}{4}\epsilon^{\mu\nu\lambda\sigma} V_{\mu\nu} W_{\lambda\sigma} - V^{\alpha\dot{\alpha}} W_{\alpha\dot{\alpha}}$$
- Complete PS decomposition of W :

$$\frac{m}{4!}\phi^2_{ijkl} = -m\{\phi_\mu{}^\nu \phi_\nu{}^\mu + \phi_{\mu\nu(s)}^{\alpha\dot{\alpha}} \bar{\phi}^{\mu\nu(s)}_{\alpha\dot{\alpha}} + \vec{\phi}_{\mu(R)}^{\ \nu} . \vec{\phi}_{\nu(R)}^{\ \mu} + \vec{\phi}_{\mu(L)}^{\ \nu} . \vec{\phi}_{\nu(L)}^{\ \mu} + \frac{1}{2}\tilde{\phi}^{\mu\nu\alpha\dot{\alpha}}_{(a)} \phi^{(a)}_{\mu\nu\alpha\dot{\alpha}} - \phi^2\}$$

$$\frac{M}{5!}\Sigma_{ijklm}\overline{\Sigma}_{ijklm} = M\{\widetilde{\Sigma}^{\mu\nu}_{(a)} \overline{\Sigma}^{(a)}_{\mu\nu} + 2\Sigma_\nu{}^{\mu\alpha\dot{\alpha}} \overline{\Sigma}_\mu{}^\nu{}_{\alpha\dot{\alpha}} + (\vec{\overline{\Sigma}}^{(s)}_{\mu\nu(R)} . \vec{\Sigma}^{\mu\nu(s)}_{(R)} + \vec{\Sigma}^{(s)}_{\mu\nu(L)} . \vec{\overline{\Sigma}}^{\mu\nu}_{(s)(L)})\}$$

$$\begin{aligned}
\frac{1}{4!}\lambda\phi^3 \quad &= \quad \lambda[-\frac{2}{3} i \phi_\mu{}^\nu \phi_\nu{}^\lambda \phi_\lambda{}^\mu - 2i(\phi_\mu{}^\nu \phi^{\alpha\dot{\alpha}}_{\nu\lambda(s)} \bar{\phi}^{\lambda\mu(s)}_{\ \alpha\dot{\alpha}}) \\
&- \quad 2i\{\phi_\mu{}^\nu (\vec{\phi}_{\nu(R)}^{\ \lambda} . \vec{\phi}_{\lambda(R)}^{\ \mu} + \vec{\phi}_{\nu(L)}^{\ \lambda} . \vec{\phi}_{\lambda(L)}^{\ \mu})\} \\
&+ \quad \{(\tilde{\phi}^{\mu\nu\alpha\dot{\alpha}}_{(a)} (\phi^{\dot{\beta}}_{\mu\lambda(s)\alpha} \phi_{\nu(R)\dot{\alpha}\dot{\beta}}^{\ \lambda} - \phi^{\beta}_{\mu\lambda(s)\dot{\alpha}} \phi_{\nu(L)\alpha\beta}^{\ \lambda})) \\
&+ \quad (\phi^{\alpha\dot{\alpha}}_{\mu\nu(a)} (-\phi^{\mu\lambda\ \dot{\beta}}_{\alpha} \phi_{\lambda(R)\dot{\alpha}\dot{\beta}}^{\ \nu} + \phi^{\mu\lambda(s)\beta}_{\dot{\alpha}} \phi_{\lambda(L)\alpha\beta}^{\ \nu}))\} \\
&+ \quad \sqrt{2}\{\phi^{\ \alpha\dot{\alpha}}_{\nu\lambda(s)} (\phi^{\lambda\mu\ \dot{\beta}}_{(s)\ \alpha} \phi_{\mu(R)\dot{\alpha}\dot{\beta}}^{\ \nu} + \phi^{\lambda\mu\ \beta}_{(s)\ \dot{\alpha}} \phi_{\mu(L)\alpha\beta}^{\ \nu})\} \\
&- \quad \phi\{(\vec{\phi}_{\nu(R)}^{\ \lambda} . \vec{\phi}_{\lambda(R)}^{\ \nu} - \vec{\phi}_{\nu(L)}^{\ \lambda} . \vec{\phi}_{\lambda(L)}^{\ \nu})\} \\
&- \quad \frac{1}{\sqrt{2}}\{\phi^{\ \alpha\dot{\alpha}}_{\mu\nu(a)} (\phi^{\nu\lambda\ \dot{\beta}}_{(a)\ \alpha} \phi_{\lambda(R)\dot{\alpha}\dot{\beta}}^{\ \mu} + \phi^{\nu\lambda\ \beta}_{(a)\ \dot{\alpha}} \phi_{\lambda(L)\alpha\beta}^{\ \mu})\} \\
&+ \quad \frac{\sqrt{2}}{3}\{\phi_{\mu(R)}^{\ \nu\ \dot{\alpha}\dot{\beta}} \phi_\nu{}^{\lambda\ \dot{\gamma}}_{\ (R)\dot{\beta}} \phi_{\lambda(R)\dot{\gamma}\dot{\alpha}}^{\ \mu} + \phi_{\mu(L)}^{\ \nu\alpha\beta} \phi_\nu{}^{\lambda\ \gamma}_{\ (L)\beta} \phi_{\lambda(L)\gamma\alpha}^{\ \mu}\}]
\end{aligned}$$

$$\begin{aligned}
\frac{1}{4!}\bar{\gamma}\phi\overline{\Sigma}H \quad &= \quad \bar{\gamma}[iH_{\mu\nu(a)} \overline{\Sigma}^{\nu\lambda(a)} \phi_\lambda{}^\mu + \phi_\mu{}^\nu \overline{\Sigma}_\nu{}^{\mu\alpha\dot{\alpha}} H_{\alpha\dot{\alpha}} \\
&- \quad \frac{1}{2} H^{\alpha\dot{\alpha}} (\phi^{\mu\nu\ \dot{\beta}}_{(s)\alpha} \overline{\Sigma}_{\mu\nu(s)\dot{\alpha}\dot{\beta}(R)} + \phi^{(s)\beta}_{\mu\nu\ \dot{\alpha}} \overline{\Sigma}^{\mu\nu(s)}_{\alpha\beta(L)})\} \\
&- \quad \frac{1}{\sqrt{2}}\{\tilde{H}^{\mu\nu(a)} \overline{\Sigma}^{\lambda\alpha\dot{\alpha}}_\nu \phi_{\mu\lambda(s)\alpha\dot{\alpha}} + H_{\mu\nu(a)} \overline{\Sigma}_\lambda{}^{\nu\alpha\dot{\alpha}} \bar{\phi}^{\mu\lambda(s)}_{\alpha\dot{\alpha}}\}
\end{aligned}$$

$$
\begin{aligned}
&- \frac{i}{\sqrt{2}}\{H^{\alpha\dot{\alpha}}(\overline{\Sigma}_{\mu\ \alpha}^{\ \nu\ \dot{\beta}}\phi_{\ \nu(R)\dot{\alpha}\dot{\beta}}^{\mu} - \overline{\Sigma}_{\mu\ \dot{\alpha}}^{\ \nu\ \beta}\phi_{\ \nu(L)\alpha\beta}^{\mu})\} \\
&+ (-\tilde{H}^{\mu\nu(a)}\vec{\phi}_{\ \nu(R)}^{\lambda}.\vec{\overline{\Sigma}}_{\mu\lambda(s)(R)} + H_{\mu\nu(a)}\vec{\phi}_{\ \lambda(L)}^{\nu}.\vec{\overline{\Sigma}}_{(L)}^{\mu\lambda(s)}) \\
&- \frac{1}{2}\tilde{\overline{\Sigma}}^{\mu\nu(a)}\phi_{\mu\nu(a)}^{\alpha\dot{\alpha}}H_{\alpha\dot{\alpha}} \\
&- iH_{\mu\nu(a)}\phi^{\nu\lambda(a)\alpha\dot{\alpha}}\overline{\Sigma}_{\lambda\ \alpha\dot{\alpha}}^{\ \mu} + \frac{1}{2}\phi\widetilde{H}^{\mu\nu(a)}\overline{\Sigma}_{\mu\nu(a)}]
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{4!}\gamma\phi\Sigma H &= \gamma[-iH_{\mu\nu(a)}\Sigma^{\nu\lambda(a)}\phi_{\lambda}^{\ \mu} + \phi_{\mu}^{\ \nu}\Sigma_{\nu}^{\ \mu\alpha\dot{\alpha}}H_{\alpha\dot{\alpha}} \\
&- \frac{1}{2}H^{\alpha\dot{\alpha}}(\phi_{\mu\nu\ \alpha}^{(s)\dot{\beta}}\Sigma_{\dot{\alpha}\dot{\beta}(R)}^{\mu\nu(s)} + \phi_{\dot{\alpha}}^{\mu\nu\ \beta}\Sigma_{\mu\nu(s)\alpha\beta(L)}) \\
&- \frac{1}{\sqrt{2}}\{\tilde{H}^{\mu\nu(a)}\Sigma_{\nu}^{\lambda\alpha\dot{\alpha}}\phi_{\mu\lambda(s)\alpha\dot{\alpha}} + H_{\mu\nu(a)}\Sigma_{\lambda}^{\ \nu\alpha\dot{\alpha}}\bar{\phi}_{\alpha\dot{\alpha}}^{\mu\lambda(s)}\} \\
&+ \frac{i}{\sqrt{2}}\{H^{\alpha\dot{\alpha}}(\Sigma_{\mu\ \alpha}^{\ \nu\ \dot{\beta}}\phi_{\ \nu(R)\dot{\alpha}\dot{\beta}}^{\mu} - \Sigma_{\mu\ \dot{\alpha}}^{\ \nu\ \beta}\phi_{\ \nu(L)\alpha\beta}^{\mu})\} \\
&+ (-\tilde{H}^{\mu\nu(a)}\vec{\phi}_{\ \nu(L)}^{\lambda}.\vec{\Sigma}_{\mu\lambda}^{(s)(L)} + H_{\mu\nu}^{(a)}\vec{\phi}_{\ \lambda(R)}^{\nu}.\vec{\Sigma}_{\mu\lambda}^{(s)(R)}) \\
&- \frac{1}{2}\widetilde{\Sigma}^{\mu\nu(a)}\phi_{\mu\nu(a)}^{\alpha\dot{\alpha}}H_{\alpha\dot{\alpha}} \\
&+ iH_{\mu\nu(a)}\phi_{(a)}^{\nu\lambda\alpha\dot{\alpha}}\Sigma_{\lambda\ \alpha\dot{\alpha}}^{\ \mu} + \frac{1}{2}\phi\widetilde{H}_{(a)}^{\mu\nu}\Sigma_{\mu\nu}^{(a)}\}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\eta}{4!}\phi\Sigma\overline{\Sigma} &= \eta[2i\phi_{\mu}^{\ \nu}(\Sigma_{\nu}^{\ \lambda\alpha\dot{\alpha}}\overline{\Sigma}_{\lambda\alpha\dot{\alpha}}^{\mu} + \overline{\Sigma}_{\nu}^{\ \lambda\alpha\dot{\alpha}}\Sigma_{\lambda\ \alpha\dot{\alpha}}^{\ \mu}) \\
&+ 2i\phi_{\mu}^{\ \nu}(\vec{\overline{\Sigma}}_{\nu\lambda(s)(R)}.\vec{\Sigma}_{(s)(R)}^{\mu\lambda} + \vec{\Sigma}_{\nu\lambda(s)(L)}.\vec{\overline{\Sigma}}_{(s)(L)}^{\mu\lambda}) \\
&+ \phi(\vec{\overline{\Sigma}}_{\mu\nu(s)(R)}.\vec{\Sigma}_{(s)(R)}^{\mu\nu} - \vec{\Sigma}_{\mu\nu(s)(L)}\cdot\vec{\overline{\Sigma}}_{(s)(L)}^{\mu\nu}) \\
&+ i\sqrt{2}(-\tilde{\Sigma}^{\mu\nu(a)}\overline{\Sigma}_{\nu}^{\ \lambda\ \alpha\dot{\alpha}}\phi_{\mu\lambda(s)\alpha\dot{\alpha}} + \Sigma_{\mu\nu(a)}\overline{\Sigma}_{\lambda}^{\ \nu\ \alpha\dot{\alpha}}\phi_{\alpha\dot{\alpha}}^{\mu\lambda(s)}) \\
&+ i\sqrt{2}(\tilde{\overline{\Sigma}}^{\mu\nu(a)}\Sigma_{\nu}^{\ \lambda\ \alpha\dot{\alpha}}\phi_{\mu\lambda(s)\alpha\dot{\alpha}} - \overline{\Sigma}_{\mu\nu(a)}\Sigma_{\lambda}^{\ \nu\ \alpha\dot{\alpha}}\phi_{\alpha\dot{\alpha}}^{\mu\lambda(s)}) \\
&- 2i\Sigma_{\mu}^{\ \nu\alpha\dot{\alpha}}(\phi_{\ \nu\lambda(s)\dot{\alpha}}^{\beta}\overline{\Sigma}_{\alpha\beta(L)}^{\mu\lambda(s)} + \overline{\Sigma}_{\nu\lambda\dot{\alpha}\dot{\beta}(R)}^{(s)}\phi_{\alpha}^{\mu\lambda(s)\dot{\beta}}) \\
&- 2i\overline{\Sigma}_{\mu}^{\ \nu\alpha\dot{\alpha}}(\phi_{\ \nu\lambda(s)\alpha}^{\dot{\beta}}\Sigma_{\dot{\alpha}\dot{\beta}(R)}^{\mu\lambda(s)} + \Sigma_{\nu\lambda\alpha\beta(L)}^{(s)}\phi_{\dot{\alpha}}^{\mu\lambda(s)\beta}) \\
&- 2(\tilde{\Sigma}^{\mu\nu(a)}\vec{\phi}_{\nu(R)}^{\lambda}.\vec{\overline{\Sigma}}_{\mu\lambda(R)} + \Sigma_{\mu\nu(a)}\vec{\phi}_{\lambda(L)}^{\nu}.\vec{\overline{\Sigma}}^{\mu\lambda(L)}) \\
&+ 2(\tilde{\overline{\Sigma}}^{\mu\nu(a)}\vec{\phi}_{\nu(L)}^{\lambda}.\vec{\Sigma}_{\mu\lambda(L)} + \overline{\Sigma}_{\mu\nu(a)}\vec{\phi}_{\lambda(R)}^{\nu}.\vec{\Sigma}^{\mu\lambda(R)}) \\
&- 2\sqrt{2}(\overline{\Sigma}_{\mu}^{\ \nu\alpha\dot{\alpha}}\Sigma_{\nu\alpha}^{\lambda\dot{\beta}}\phi_{\ \lambda(R)\dot{\alpha}\dot{\beta}}^{\mu} + \Sigma_{\mu}^{\ \nu\alpha\dot{\alpha}}\overline{\Sigma}_{\nu\dot{\alpha}}^{\ \lambda\beta}\phi_{\lambda(L)\alpha\beta}^{\mu}) \\
&- \sqrt{2}(\phi_{\nu\ (R)}^{\ \mu\ \dot{\alpha}\dot{\beta}}\overline{\Sigma}_{\mu\lambda\dot{\beta}}^{(s)(R)\dot{\gamma}}\Sigma_{(R)\dot{\gamma}\dot{\alpha}}^{\nu\lambda(s)} + \phi_{\nu\ (L)}^{\ \mu\ \alpha\beta}\Sigma_{\mu\lambda\beta}^{(s)(L)\gamma}\overline{\Sigma}_{(L)\gamma\alpha}^{\nu\lambda(s)}) \\
&+ i\sqrt{2}(-\tilde{\phi}_{(a)}^{\mu\nu\alpha\dot{\alpha}}\overline{\Sigma}_{\nu\ \dot{\alpha}}^{\ \lambda\ \beta}\Sigma_{\mu\lambda\alpha\beta}^{(s)(L)} + \phi_{\mu\nu(a)}^{\alpha\dot{\alpha}}\overline{\Sigma}_{\lambda\ \alpha}^{\ \nu\ \dot{\beta}}\Sigma_{\dot{\alpha}\dot{\beta}(R)}^{\mu\lambda(s)}) \\
&+ i\sqrt{2}(\tilde{\phi}_{(a)}^{\mu\nu\alpha\dot{\alpha}}\Sigma_{\nu\ \alpha}^{\ \lambda\ \dot{\beta}}\overline{\Sigma}_{\mu\lambda\dot{\alpha}\dot{\beta}}^{(s)(R)} - \phi_{\mu\nu(a)}^{\alpha\dot{\alpha}}\Sigma_{\lambda\ \dot{\alpha}}^{\ \nu\ \beta}\overline{\Sigma}_{\dot{\alpha}\dot{\beta}(L)}^{\mu\lambda(s)})]
\end{aligned}
$$

ii) Chiral Mixed states

a)$[8,1,0](R_1,R_2) \equiv (\phi_{\bar\mu}{}^{\bar\nu}, \phi_{\bar\mu(R0)}^{\bar\nu})$

$$\mathcal{R} = \begin{pmatrix} (m-\lambda a) & -\sqrt{2}\lambda\omega \\ -\sqrt{2}\lambda\omega & m+\lambda(p-a) \end{pmatrix} \tag{1}$$

$$m_{R_\pm} = 2|\mathcal{R}_\pm| = |2m[1+(\frac{\tilde p}{2}-\tilde a) \pm \sqrt{(\frac{\tilde p}{2})^2 + 2\tilde\omega^2}]| \tag{2}$$

The corresponding eigenvectors can be found by diagonalizing the matrix $\mathcal{R}\mathcal{R}^\dagger$.

b) $[1,2,-1](\bar h_1,\bar h_2,\bar h_3,\bar h_4) \oplus [1,2,1](h_1,h_2,h_3,h_4)$
$\equiv (H_{\dot 2}^{\alpha}, \overline{\Sigma}_{\dot 2}^{(15)\alpha}, \Sigma_{\dot 2}^{(15)\alpha}, \frac{\phi_{44}^{\dot 2\alpha}}{\sqrt2}) \oplus (H_{\alpha\dot 1}, \overline{\Sigma}_{\alpha\dot 1}^{(15)}, \Sigma_{\alpha\dot 1}^{(15)}, \frac{\phi_{\alpha}^{44\dot 1}}{\sqrt2})$

$$\mathcal{H} = \begin{pmatrix} -M_H & +\overline{\gamma}\sqrt{3}(\omega-a) & -\gamma\sqrt{3}(\omega+a) & -\bar\gamma\bar\sigma \\ -\overline{\gamma}\sqrt{3}(\omega+a) & 0 & -(2M+4\eta(a+\omega)) & 0 \\ \gamma\sqrt{3}(\omega-a) & -(2M+4\eta(a-\omega)) & 0 & -2\eta\overline{\sigma}\sqrt{3} \\ -\sigma\gamma & -2\eta\sigma\sqrt{3} & 0 & -2m+6\lambda(\omega-a) \end{pmatrix}$$

The above matrix is to be diagonalized after imposing the fine tuning condition $Det\mathcal{H}=0$ to keep one pair of doublets light.

c) $[\bar 3,1,\frac{2}{3}](\bar t_1,\bar t_2,\bar t_3,\bar t_4,\bar t_5) \oplus [3,1,-\frac{2}{3}](t_1,t_2,t_3,t_4,t_5)$
$\equiv (H^{\bar\mu 4}, \overline{\Sigma}_{(a)}^{\bar\mu 4}, \Sigma_{(a)}^{\bar\mu 4}, \Sigma_{R0}^{\bar\mu 4}, \phi_{4(R+)}^{\bar\mu}) \oplus (H_{\bar\mu 4}, \overline{\Sigma}_{(a)\bar\mu 4}, \Sigma_{\bar\mu 4(a)}, \overline{\Sigma}_{\bar\mu 4(R0)}, \phi_{\bar\mu(R-)}^{4})$

$$\mathcal{T} = \begin{pmatrix} M_H & \overline{\gamma}(a+p) & \gamma(p-a) & 2\sqrt{2}i\omega\bar\gamma & i\bar\sigma\bar\gamma \\ \bar\gamma(p-a) & 0 & 2M & 0 & 0 \\ \gamma(p+a) & 2M & 0 & 4\sqrt{2}i\omega\eta & 2i\eta\overline{\sigma} \\ -2\sqrt{2}i\omega\gamma & -4\sqrt{2}i\omega\eta & 0 & 2M+2\eta p+2\eta a & -2\sqrt{2}\eta\overline{\sigma} \\ i\sigma\gamma & 2i\eta\sigma & 0 & 2\sqrt{2}\eta\sigma & -2m-2\lambda(a+p-4\omega) \end{pmatrix}$$

iii) Mixed gauge chiral .

a)$[1,1,0](G_1,G_2,G_3,G_4,G_5,G_6) \equiv (\phi, \phi^{(15)}, \phi_{(R0)}^{(15)}, \frac{\Sigma_{(R-)}^{44}}{\sqrt2}, \frac{\overline{\Sigma}_{44((R+)}}{\sqrt2}, \frac{\sqrt2\lambda^{(R0)}-\sqrt3\lambda^{(15)}}{\sqrt5})$

$$\mathcal{G} = 2\begin{pmatrix} m & 0 & \sqrt6\lambda\omega & \frac{i\eta\overline{\sigma}}{\sqrt2} & \frac{-i\eta\sigma}{\sqrt2} & 0 \\ 0 & m+2\lambda a & 2\sqrt2\lambda\omega & i\eta\overline{\sigma}\sqrt{\frac32} & -i\eta\sigma\sqrt{\frac32} & 0 \\ \sqrt6\lambda\omega & 2\sqrt2\lambda\omega & m+\lambda(p+2a) & -i\eta\sqrt3\overline{\sigma} & i\sqrt3\eta\sigma & 0 \\ \frac{i\eta\overline{\sigma}}{\sqrt2} & i\eta\overline{\sigma}\sqrt{\frac32} & -i\eta\sqrt3\overline{\sigma} & 0 & M+\eta(p+3a-6\omega) & \frac{\sqrt5 g\sigma^*}{2} \\ \frac{-i\eta\sigma}{\sqrt2} & -i\eta\sigma\sqrt{\frac32} & i\eta\sqrt3\sigma & M+\eta(p+3a-6\omega) & 0 & \frac{\sqrt5 g\overline{\sigma}^*}{2} \\ 0 & 0 & 0 & \frac{\sqrt5 g\sigma^*}{2} & \frac{\sqrt5 g\overline{\sigma}^*}{2} & 0 \end{pmatrix}$$

Field$[SU(3),SU(2),Y]$	$PS \quad Fields$	Mass
$A[1,1,4],\bar{A}[1,1,-4]$	$\frac{\Sigma^{44}_{(R+)}}{\sqrt{2}},\frac{\overline{\Sigma}_{44(R-)}}{\sqrt{2}}$	$2(M+\eta(p+3a+6\omega))$
$C_1[8,2,1],\bar{C}_1[(8,2,-1]$	$\Sigma_{\bar{\nu}\alpha\dot{1}}^{\ \bar{\lambda}},\overline{\Sigma}_{\bar{\nu}\alpha\dot{2}}^{\ \bar{\lambda}}$	$2(-M+\eta(a+\omega))$
$C_2[8,2,1],\bar{C}_2[(8,2,-1]$	$\overline{\Sigma}_{\bar{\nu}\alpha\dot{1}}^{\ \lambda},\Sigma_{\bar{\nu}\alpha\dot{2}}^{\ \bar{\lambda}}$	$2(-M+\eta(a-\omega))$
$D_1[3,2,\frac{7}{3}],\bar{D}_1[(\bar{3},2,-\frac{7}{3}]$	$\overline{\Sigma}_{\bar{\nu}\alpha\dot{1}}^{\ 4},\Sigma_{4\alpha\dot{2}}^{\ \bar{\nu}}$	$2(M+\eta(a+\omega))$
$D_2[3,2,\frac{7}{3}],\bar{D}_2[(\bar{3},2,-\frac{7}{3}]$	$\Sigma_{\bar{\nu}\alpha\dot{1}}^{\ 4},\overline{\Sigma}_{4\alpha\dot{2}}^{\ \bar{\nu}}$	$2(M+\eta(a+3\omega))$
$E_1[3,2,\frac{1}{3}],\bar{E}_1[(\bar{3},2,-\frac{1}{3}]$	$\overline{\Sigma}_{\bar{\nu}\alpha\dot{2}}^{\ 4},\Sigma_{4\alpha\dot{1}}^{\ \bar{\nu}}$	$-2(M+\eta(a-\omega))$
$K[3,1,-\frac{8}{3}],\bar{K}[(\bar{3},1,\frac{8}{3}]$	$\overline{\Sigma}_{\bar{\nu}4(R-)},\Sigma^{\bar{\nu}4}_{(R+)}$	$2(M+\eta(a+p+2\omega))$
$L[6,1,\frac{2}{3}],\bar{L}[(\bar{6},1,-\frac{2}{3}]$	$(\overline{\Sigma}'^{(R0)}_{\bar{\mu}\bar{\nu}},\Sigma'^{\bar{\mu}\bar{\nu}}_{(R0)})_{\bar{\mu}\leq\bar{\nu}}$	$2(M+\eta(p-a))$
	$\overline{\Sigma}'_{\bar{\mu}\bar{\nu}}=\overline{\Sigma}_{\bar{\mu}\bar{\nu}},\bar{\mu}\neq\bar{\nu}$	
	$\overline{\Sigma}'_{\bar{\mu}\bar{\mu}}=\frac{\overline{\Sigma}_{\bar{\mu}\bar{\mu}}}{\sqrt{2}}$	
$M[6,1,\frac{8}{3}],\overline{M}[(\bar{6},1,-\frac{8}{3}]$	$(\overline{\Sigma}'^{(R+)}_{\bar{\mu}\bar{\nu}(R+)},\Sigma'^{\bar{\mu}\bar{\nu}}_{(R-)})_{\bar{\mu}\leq\bar{\nu}}$	$2(M+\eta(p-a+2\omega))$
$N[6,1,-\frac{4}{3}],\bar{N}[(\bar{6},1,\frac{4}{3}]$	$(\overline{\Sigma}'^{(R-)}_{\bar{\mu}\bar{\nu}},\Sigma'^{\bar{\mu}\bar{\nu}}_{(R+)})_{\bar{\mu}\leq\bar{\nu}}$	$2(M+\eta(p-a-2\omega))$
$O[1,3,-2],\bar{O}[(1,3,+2]$	$\frac{\vec{\Sigma}_{44(L)}}{\sqrt{2}},\frac{\vec{\overline{\Sigma}}^{44}_{(L)}}{\sqrt{2}}$	$2(M+\eta(3a-p))$
$P[3,3,-\frac{2}{3}],\bar{P}[\bar{3},3,\frac{2}{3}]$	$\vec{\Sigma}_{\bar{\mu}4(L)},\vec{\overline{\Sigma}}^{\bar{\mu}4}_{(L)}$	$2(M+\eta(a-p))$
$W[6,3,\frac{2}{3}],\overline{W}[(\bar{6},3,-\frac{2}{3}]$	$\vec{\Sigma}'_{\bar{\mu}\bar{\nu}(L)},\vec{\overline{\Sigma}}^{\bar{\mu}\bar{\nu}}_{(L)}$	$2(M-\eta(a+p))$
$I[3,1,\frac{10}{3}],\bar{I}[(\bar{3},1,-\frac{10}{3}]$	$\phi^{4}{}_{\bar{\nu}(R+)},\phi_{4(R-)}^{\bar{\nu}}$	$-2(m+\lambda(p+a+4\omega))$
$S[1,3,0]$	$\vec{\phi}^{(15)}_{(L)}$	$2(m+\lambda(2a-p))$
$Q[8,3,0]$	$\vec{\phi}_{\bar{\mu}(L)}^{\ \bar{\nu}}$	$2(m-\lambda(a+p))$
$U[3,3,\frac{4}{3}],\bar{U}[\bar{3},3,-\frac{4}{3}]$	$\vec{\phi}_{\bar{\mu}(L)}^{\ 4},\vec{\phi}_{4(L)}^{\ \bar{\mu}}$	$-2(m-\lambda(p-a))$
$V[1,2,-3],\bar{V}[1,2,3]$	$\frac{\phi_{44\alpha\dot{2}}}{\sqrt{2}},\frac{\phi^{44}_{\ \alpha\dot{1}}}{\sqrt{2}}$	$2(m+3\lambda(a+\omega))$
$B[6,2,\frac{5}{3}],\bar{B}[(\bar{6},2,-\frac{5}{3}]$	$(\phi'_{\bar{\mu}\bar{\nu}\alpha\dot{1}},\phi'^{\bar{\mu}\bar{\nu}}_{\ \alpha\dot{2}})_{\bar{\mu}\leq\bar{\nu}}$	$2(m+\lambda(\omega-a))$
$Y[6,2,-\frac{1}{3}],\bar{Y}[(\bar{6},2,\frac{1}{3}]$	$(\phi'_{\bar{\mu}\bar{\nu}\alpha\dot{2}},\phi'^{\bar{\mu}\bar{\nu}}_{\ \alpha\dot{1}})_{\bar{\mu}\leq\bar{\nu}}$	$2(m-\lambda(a+\omega))$
$Z[8,1,2],\bar{Z}[8,1,-2]$	$\phi^{\bar{\nu}}{}_{\bar{\mu}(R+)}\phi_{\bar{\mu}(R-)}^{\ \bar{\nu}}$	$2(m+\lambda(p-a))$

Table 1: **Spectra of 26 $G_{123}-$types of MSGUT multiplets :**
i) Masses of the unmixed states in terms of the superheavy vevs . The $SU(2)_L$ contraction order is always $\bar{F}^{\alpha}F_{\alpha}$. The primed fields defined for $SU(3)_c$ sextets maintain unit norm. The absolute value of the expressions in the column "Mass" is understood.

b) $[\bar{3},2,-\frac{1}{3}](\bar{E}_2,\bar{E}_3,\bar{E}_4,\bar{E}_5)\oplus[3,2,\frac{1}{3}](E_2,E_3,E_4,E_5)$
$\equiv(\overline{\Sigma}^{\bar{\mu}}_{4\alpha\dot{1}},\phi^{\bar{\mu}4}_{(s)\alpha\dot{2}},\phi^{(a)\bar{\mu}4}_{\alpha\dot{2}},\lambda^{\bar{\mu}4}_{\alpha\dot{2}})\oplus(\Sigma^{4}_{\bar{\mu}\alpha\dot{2}},\phi^{(s)}_{\bar{\mu}4\alpha\dot{1}},\phi^{(a)}_{\bar{\mu}4\alpha\dot{1}},\lambda_{\bar{\mu}\alpha\dot{1}})$

$$\mathcal{E}=\begin{pmatrix} -2(M+\eta(a-3\omega)) & -2\sqrt{2}i\eta\sigma & 2i\eta\sigma & ig\sqrt{2}\overline{\sigma}^* \\ 2i\sqrt{2}\eta\overline{\sigma} & -2(m+\lambda(a-\omega)) & -2\sqrt{2}\lambda\omega & 2g(a^*-\omega^*) \\ -2i\eta\overline{\sigma} & -2\sqrt{2}\lambda\omega & -2(m-\lambda\omega) & \sqrt{2}g(\omega^*-p^*) \\ -ig\sqrt{2}\sigma^* & 2g(a^*-\omega^*) & g\sqrt{2}(\omega^*-p^*) & 0 \end{pmatrix}$$

c) $[1,1,-2](\bar{F}_1,\bar{F}_2,\bar{F}_3)\oplus[1,1,2](F_1,F_2,F_3)$
$\equiv(\overline{\Sigma}_{44(R0)},\phi^{(15)}_{(R-)},\lambda_{(R-)})\oplus(\Sigma^{44}_{(R0)},\phi^{(15)}_{(R+)},\lambda_{(R+)})$.

$$\mathcal{F}=\begin{pmatrix} 2(M+\eta(p+3a)) & -2i\sqrt{3}\eta\sigma & -g\sqrt{2}\overline{\sigma}^* \\ 2i\sqrt{3}\eta\overline{\sigma} & 2(m+\lambda(p+2a)) & \sqrt{24}ig\omega^*) \\ -g\sqrt{2}\sigma^* & -\sqrt{24}ig\omega^* & 0 \end{pmatrix}$$

d) $[\bar{3},1,-\frac{4}{3}](\bar{J}_1,\bar{J}_2,\bar{J}_3,\bar{J}_4)\oplus[3,1,\frac{4}{3}](J_1,J_2,J_3,J_4)$
$\equiv(\Sigma^{\bar{\mu}4}_{(R-)},\phi^{\bar{\mu}}_{4},\phi_4{}^{\bar{\mu}(R0)},\lambda_4{}^{\bar{\mu}})\oplus(\overline{\Sigma}_{\bar{\mu}4(R+)},\phi^4{}_{\bar{\mu}},\phi_{\bar{\mu}(R0)}^{\ 4},\lambda^4_{\bar{\mu}})$

$$\mathcal{J}=\begin{pmatrix} 2(M+\eta(a+p-2\omega)) & -2\eta\overline{\sigma} & 2\sqrt{2}\eta\overline{\sigma} & -ig\sqrt{2}\sigma^* \\ 2\eta\sigma & -2(m+\lambda a) & -2\sqrt{2}\lambda\omega & -2ig\sqrt{2}a^* \\ -2\sqrt{2}\eta\sigma & -2\sqrt{2}\lambda\omega & -2(m+\lambda(a+p)) & -4ig\omega^* \\ -ig\sqrt{2}\overline{\sigma}^* & 2\sqrt{2}iga^* & 4ig\omega^* & 0 \end{pmatrix} \quad (3)$$

e) $[3,2,\frac{5}{3}](\bar{X}_1,\bar{X}_2,\bar{X}_3)\oplus[3,2,-\frac{5}{3}](X_1,X_2,X_3)$
$\equiv(\phi^{(s)\bar{\mu}4}_{\alpha\dot{1}},\phi^{(a)\bar{\mu}4}_{\alpha\dot{1}},\lambda^{\bar{\mu}4}_{\alpha\dot{1}})\oplus(\phi^{(s)}_{\bar{\mu}4\alpha\dot{2}},\phi^{(a)}_{\bar{\mu}4\alpha\dot{2}},\lambda_{\bar{\mu}4\alpha\dot{2}})$

$$\mathcal{X}=\begin{pmatrix} 2(m+\lambda(a+\omega)) & -2\sqrt{2}\lambda\omega & -2g(a^*+\omega^*) \\ -2\sqrt{2}\lambda\omega & 2(m+\lambda\omega) & \sqrt{2}g(\omega^*+p^*) \\ -2g(a^*+\omega^*) & \sqrt{2}g(\omega^*+p^*) & 0 \end{pmatrix}$$

CONSISTENCY CHECKS :

AT POINTS OF SPECIAL SYMMETRY MASSES RESPECT ENHANCED SYMMETRY ! Checked explicitly for $G_{PS}, SU(5), SU(5)\times U(1)$ cases.

!! : ph-0405300 XXX

RG Analysis

- Superheavy Threshold corrections to 2-loop gauge evolution formulae :

  Weinberg(1980), Hall(1981)

- **Gauge running**

$$\frac{1}{\alpha_i(M_S)} = \frac{1}{\alpha_G(M_X)} + 8\pi b_i ln\frac{M_X}{M_S} + 4\pi\sum_j \frac{b_{ij}}{b_j} lnX_j - 4\pi\lambda_i(M_X)$$

- 2-Loops

$$X_j = 1 + 8\pi b_j \alpha_G(M_X^0) ln\frac{M_X^0}{M_S}$$

$M_X^0, \alpha_G(M_X^0)$ determined by 1-loop.
*

$$\begin{aligned}
\Delta^{(2-loop)}(lnM_X) &= -\frac{1}{10b_1 + 6b_2 - 16b_3}\sum_j [\frac{5b_{1j} + 3b_{2j} - 8b_{3j}}{b_j} lnX_j] \\
\Delta^{(2-loop)}(sin^2\theta_w(M_S)) &= -\frac{10\pi\alpha(M_S)}{(5b_1 + 3b_2 - 8b_3)}\sum_{ijkl}\epsilon_{ijk}(b_i - b_j)\frac{b_{kl}}{b_l} lnX_l
\end{aligned}$$

* Global Averages

$$\alpha_1^{-1}(M_S) = 57.45 \quad ; \quad \alpha_2^{-1}(M_S) = 30.8 \quad ;; \quad \alpha_3^{-1}(M_S) = 11.04$$
$$\alpha_G^0(M_X)^{-1} = 25.6 \quad ; \quad M_X^0 = 10^{16.25} GeV \quad ; \quad M_S = 1TeV$$

* Numerical values: **Small !**

$$\mathbf{\Delta^{2-loop}(log_{10}\frac{M_X}{M_S}) = -.08} \quad ; \quad \mathbf{\Delta^{2-loop}(sin^2\theta_W(M_S)) = .0026}$$

- **Superheavy threshold effects**

$$\lambda_i(\mu) = -\frac{2}{21}(b_{iV}+b_{iGB}) + 2(b_{iV}+b_{iGB})ln\frac{M_V}{\mu} + 2b_{iS}ln\frac{M_V}{\mu} + 2b_{iF}ln\frac{M_F}{\mu}$$

  * Corrections to $M_X^0, Sin^2(\theta_W)$:

$$\begin{aligned}
\Delta^{(th)}(lnM_X) &= \frac{5\lambda_1(M_X^0)+3\lambda_2(M_X^0)-8\lambda_3(M_X^0)}{10b_1+6b_2-16b_3} \\
\boldsymbol{\Delta^{(th)}}(\mathbf{Log_{10}M_X}) &= \mathbf{.0217} + \mathbf{.0167}\sum_{\mathbf{M'}}(\mathbf{5\bar{b}_1'} + \mathbf{3\bar{b}_2'} - \mathbf{8\bar{b}_3'})\mathbf{Log_{10}}\frac{\mathbf{M'}}{\mathbf{M_X^0}} \\
\boldsymbol{\Delta^{(th)}}(\mathbf{sin^2}\boldsymbol{\theta}_\mathbf{W}(\mathbf{M_S})) &= \frac{10\pi\alpha(M_S)}{(5b_1+3b_2-8b_3)}\sum_{ijk}\epsilon_{ijk}(b_i-b_j)\lambda_k(M_X^0) \\
&= \mathbf{.00004} - \mathbf{.00024}\sum_{\mathbf{M'}}(\mathbf{4\bar{b}_1'} - \mathbf{9.6\bar{b}_2'} + \mathbf{5.6\bar{b}_3'})\mathbf{Log_{10}}\frac{\mathbf{M'}}{\mathbf{M_X^0}}
\end{aligned}$$

  $\bar{b}_i = 16\pi^2 b_i$; $\bar{b}$: Vector, complex scalar, Weyl : $-11S(R)/3, S(R)/3, 2S(R)/3$

- $S_W = 4S_1 - 9.6S_2 + 5.6S_3; \quad S_X = 5S_1 + 3S_2 - 8S_3 \, : \, \sim 10^{1-2}$

  * $lnM'/M_X \sim 1$

- 1989 : Dixit, Sher : *Futility of High Precision SO(10) Calculations* !!

  ***!! Cancellations possible and likely ! No reason M_X at spectrum edge !**

  * Calculated spectrum with controlled radiative corrections and scannable parameters : MSGUT !!

- 1st complete threshold effects calculation ?!

Scan of Threshold corrections as $f(\xi)$

- m_{210} sets overall scale, M_{126}/m_{210} sensitive control parameter of Mass spectrum.($\xi = \lambda M/\eta m$)
- "Insensitive"(Logarithmic) parameters $(\lambda, \eta, \gamma, \bar{\gamma})$:fixed randomly, insensitivity checked.($\lambda = 0.12 \quad ; \quad \eta = 0.21 \quad ; \quad \gamma = 0.23 \quad ; \quad \bar{\gamma} = 0.35$)
- Real superpotential parameters, cubic $\Rightarrow$ 1 real, 2 complex(conjugate) solutions.

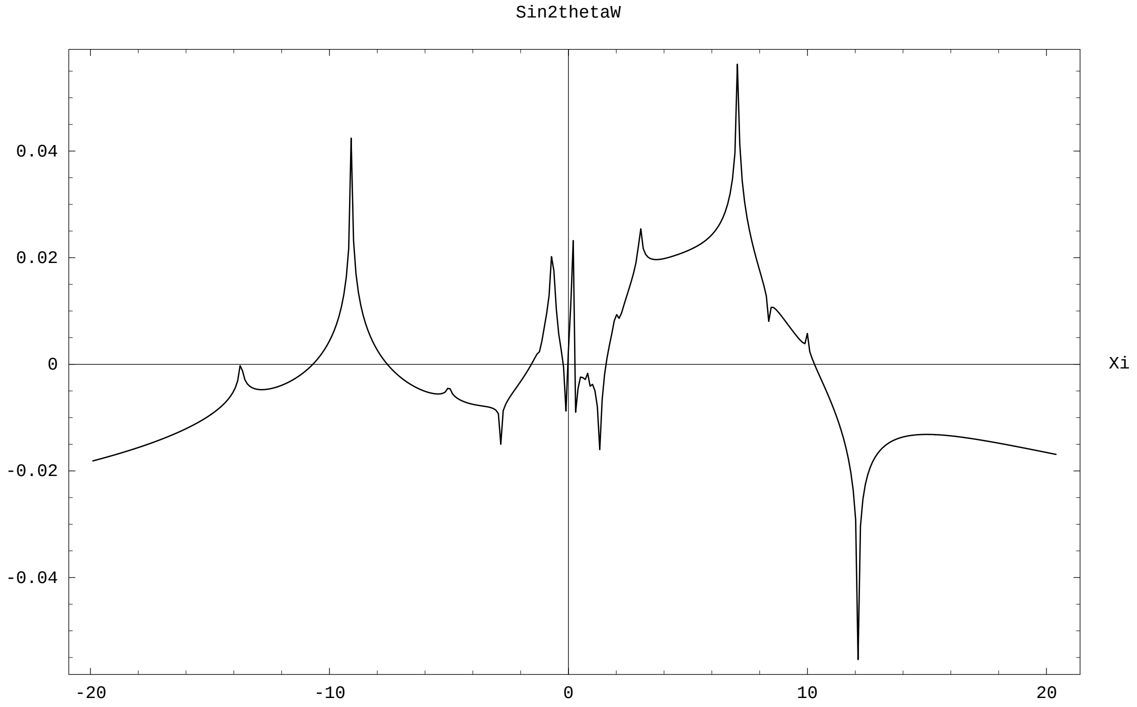

Figure 1: Plot of the threshold corrections to $Sin^2\theta_w$ vs ξ for real ξ : real solution for x.

- Fig.1 : $\Delta(Sin^2(\theta_W)) \leq 10\%$!

  * Except very near Special values of ξ (still $\leq 25\%$).
- Special points : $\xi = -5, 10$ (SU(5)) ; $\xi = 3$ (G_{LR}) ; $\xi = -2/3$ ($(SU(5) \times U(1))_{flipped}$; Others Stronger $(\xi = -9.5, -3, 7, 12)$? ?

Field$[SU(3),SU(2),Y]$	$\{S_3,S_2,S_1\}$	S_W	S_X
$A[1,1,4]$	$\{0,0,12/5\}$	9.6	12
$B[6,2,5/3]$	$\{5,3,5\}$	19.2	-6
$C[8,2,1]$	$\{6,4,12/5\}$	4.8	-24
$D[3,2,7/3]$	$\{1,3/2,49/10\}$	10.8	21
$E[3,2,1/3]$	$\{1,3/2,1/10\}$	-8.4	-3
$F[1,1,2]$	$\{0,0,3/5\}$	2.4	3
$G[1,1,0]$	$\{0,0,0\}$	0	0
$h[1,2,1]$	$\{0,1/2,3/10\}$	-3.6	3
$I[3,1,10/3]$	$\{1/2,0,5\}$	22.8	21
$J[3,1,4/3]$	$\{1/2,0,4/5\}$	6	0
$K[3,1,8/3]$	$\{1/2,0,16/5\}$	15.6	12
$L[6,1,2/3]$	$\{5/2,0,2/5\}$	15.6	-18
$M[6,1,8/3]$	$\{5/2,0,32/5\}$	39.6	12
$N[6,1,4/3]$	$\{5/2,0,8/5\}$	20.4	-12
$O[1,3,2]$	$\{0,2,9/5\}$	-12	15
$P[3,3,2/3]$	$\{3/2,6,3/5\}$	-46.8	9
$Q[8,3,0]$	$\{9,16,0\}$	-103.2	-24
$R[8,1,0]$	$\{3,0,0\}$	16.8	-24
$S[1,3,0]$	$\{0,2,0\}$	-19.2	6
$t[3,1,2/3]$	$\{1/2,0,1/5\}$	3.6	-3
$U[3,3,4/3]$	$\{3/2,6,12/5\}$	-39.6	18
$V[1,2,3]$	$\{0,1/2,27/10\}$	6	15
$W[6,3,2/3]$	$\{15/2,12,6/5\}$	-68.4	-18
$X[3,2,5/3]$	$\{1,3/2,5/2\}$	1.2	9
$Y[6,2,1/3]$	$\{5,3,1/5\}$	0	-30
$Z[8,1,2]$	$\{3,0,24/5\}$	36	0

Table 2: Index values for the 26 different chiral multiplet types (used in the threshold corrections). Except for Q,R,S all other reps come in complex pairs.$S_W = 4S_1 - 9.6S_2 + 5.6S_3, S_X = 5S_1 + 3S_2 - 8S_3$ are the combinations that enter the threshold corrections to $Sin^2\theta_W$ and to $Log_{10}M_X$

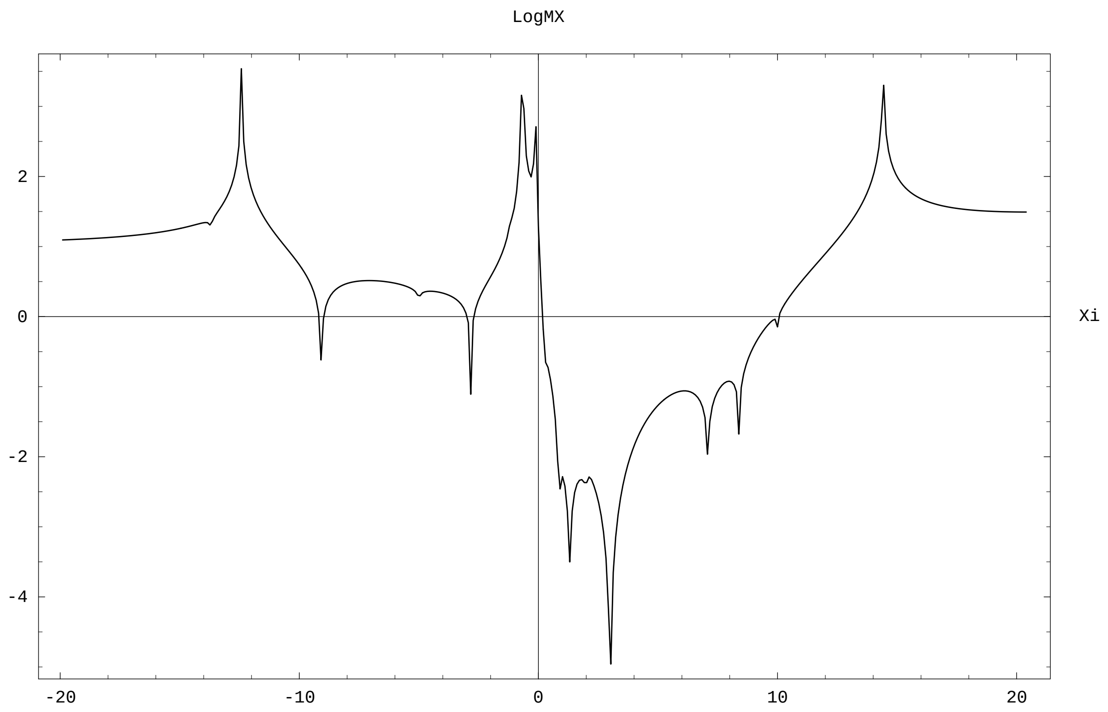

Figure 2: Plot of the threshold corrected $Log_{10} M_X/M_X^0$ vs ξ for real ξ : real solution for x.

- Fig. 2: $|Log_{10}(M_X/M_X^0)| \leq 1.5$ except near Special points (Phenomenology ?).

  * Peak Narrowness $\Rightarrow$ Corrections generically small ! : From Futility to Precision !

  *Special Points $\xi = -12, -9, -3, -1, -2/3, 1, 3, 7, 8, 14$??

  * τ_p rules out some ξ already ?

- Small corrections, slow variation generic to complex cases : far from specials. Complex ξ corrections smaller and more regular. (generic values outside special symmetry point neighbourhoods)
- Contrary to expectations precision RG analysis of the SO(10) MSGUT NOT futile :

  1-loop O(1/alpha)>> 2-loop/1-loop threshold O(1)

  generically maintained : GU picture stable !
- RG analysis , ξ scan : tomograph of SO(10) parameter space.

$d=5$ Operators for B,L violation

- B+L violation via Higgsino exchange :

$$
\begin{aligned}
W_{FM} &= 2\sqrt{2}h'_{AB}[\bar{t}_1(\epsilon\bar{u}_A\bar{d}_B + Q_A L_B) + t_1(\frac{\epsilon}{2}Q_A Q_B + \bar{u}_A\bar{e}_B - \bar{d}_A\bar{\nu}_B)] \\
&+ 4\sqrt{2}f'_{AB}[t_2(\frac{\epsilon}{2}Q_A Q_B - \bar{u}_A\bar{e}_B + \bar{\nu}_A\bar{d}_B) + \bar{t}_2(Q_A L_B - \epsilon\bar{u}_A\bar{d}_B)] \\
&- 2it_4(\bar{d}_A\bar{\nu}_B + \bar{u}_A\bar{e}_B)] + ...
\end{aligned}
$$

- **Emerging** $M_\nu, \Delta B \neq 0$ **connection**

  $*\{\bar{t}_{(1)},\bar{t}_{(2)}\}$, $\{t_{(1)},t_{(2)},t_{(4)}\}$ ($[3/\bar{3},1,\pm 2/3]$ exchange mediates B+L violation.

  $\{t_{(2)},\bar{t}_{(2)}\} \in \overline{\Sigma}(6,1,1)$,**Babu,Pati,Wilczek(1998)**

  $t_{(4)} \in \overline{\Sigma}(10,1,3)$ New ! CSA, Girdhar hep-ph0204097

$$
W_{eff}^{\Delta B \neq = 0} = L_{ABCD}(\frac{1}{2}\epsilon Q_A Q_B Q_C L_D) + R_{ABCD}(\epsilon\bar{e}_A\bar{u}_B\bar{u}_C\bar{d}_D)
$$

$$
L_{ABCD} = \mathcal{S}_1{}^1 h_{AB}h_{CD} + \mathcal{S}_1{}^2 h_{AB}f_{CD} + \mathcal{S}_2{}^1 f_{AB}h_{CD} + \mathcal{S}_2{}^2 f_{AB}f_{CD}
$$

$$
\begin{aligned}
R_{ABCD} &= \mathcal{S}_1{}^1 h_{AB}h_{CD} - \mathcal{S}_1{}^2 h_{AB}f_{CD} - \mathcal{S}_2{}^1 f_{AB}h_{CD} + \mathcal{S}_2{}^2 f_{AB}f_{CD} \\
&- i\sqrt{2}\mathcal{S}_1{}^4 f_{AB}h_{CD} + i\sqrt{2}\mathcal{S}_2{}^4 f_{AB}f_{CD}
\end{aligned}
$$

$\mathcal{S} = \mathcal{T}^{-1}$ $\quad$ $,W = \bar{t}\mathcal{T}t + ...,$

$$
h_{AB} = 2\sqrt{2}h'_{AB} \qquad f_{AB} = 4\sqrt{2}f'_{AB}
$$

Novel $d = 5, \Delta(B + L) \neq 0$ via gaugino exchange ?

* $\bar{E}_2[\bar{3}, 2, -1/3] \in \overline{\mathbf{126}}$ mixes with $SO(10)/SU(5)$ gauginos

*

$$W_{FM}^{\overline{\Sigma}} = -8\sqrt{2} f'_{AB}[\bar{E}_2 Q_B]\bar{\nu}_A = [\bar{E}_2 Q_A](f' M_{\bar{\nu}}^{-1} m^{\nu D})_{AB}\nu_B + ...$$

$$\mathcal{L}^{gauge} = \frac{g}{\sqrt{2}}[\epsilon\tilde{\bar{d}}^{*} E_5 Q + \tilde{Q}^{*} E_5 \bar{\nu} - \tilde{L}^{*} E_5 \bar{u}]$$

$\Rightarrow$

$$\mathcal{L}_{\Delta(B+L)=2} = 4g(f'(M^{\bar{\nu}})^{-1} m^{(\nu D)})_{AB} {\mathcal{E}^{-1}}_5^{\ 2} [\epsilon\tilde{\bar{d}}_C^{*} Q_C (Q_A \tilde{\nu}_B + \tilde{Q}_A \nu_B)] \Rightarrow$$

$$u_L d_L \longrightarrow \bar{d}_L + \overline{\nu_L}$$

$G_{4F} \sim (M^{\nu D} m_{Nucl}/M_X^2 M_S^2)$: Hopelessly suppressed !

Fermion Mass Formulae

$$M^{\bar\nu}_{AB} = -4i\sqrt{2}f'_{AB} < \overline{\Sigma}^{(R+)}_{44} >= 4\sqrt{2}f'_{AB}\overline{\sigma}$$

$$M^{\nu}_{AB} = 4\sqrt{2}f'_{AB} < \bar{O}^{11} >= 8if'_{AB} < \bar{O}_- >$$

$< \bar{O}_- > (\overline{10},3,1)_{\overline{\Sigma}}$ (SEESAW II)

$$m^{\bar\nu D}_{AB} = 2\sqrt{2}h'_{AB} < h^{(1)}_2 > +4i\sqrt{6}f'_{AB} < h^{(2)}_2 >$$

* Fine tuning : $Det\mathcal{H} = 0$

$$Diag(m^{(1)}_H, m^{(2)}_H, ...) = \overline{U}^T\mathcal{H}U$$

- $$M^{\nu D}_{AB} = (2\sqrt{2}h'_{AB}\alpha_1 + 4i\sqrt{6}f'_{AB}\alpha_2)v_u$$

$$< \bar{O}_- >= (\frac{i\gamma}{\sqrt{2}}\alpha_1 + 2i\sqrt{3}\eta\alpha_2)\alpha_4(\frac{v_u^2}{M_O})$$

$\alpha_i = U_{i1}, \bar\alpha_i = \bar{U}_{i1}$
$M_O = 2(M + \eta(3a - p))$.

$$\begin{aligned} M^d &= (2\sqrt{2}h'\bar\alpha_1 - 4\sqrt{\frac{2}{3}}if'\bar\alpha_2)v_d \\ M^u &= (2\sqrt{2}h'\alpha_1 - 4\sqrt{\frac{2}{3}}if'\alpha_2)v_u \\ M^l &= (2\sqrt{2}h'\bar\alpha_1 + 4\sqrt{6}if'\bar\alpha_2)v_d \end{aligned} \quad (4)$$

Ready to use !!

OUTLOOK

- $\surd$ Complete superheavy spectrum and couplings of MSGUT
- $Sin^2\theta_W, M_X$ threshold Corrections Scanned $f(\xi)$
- Precision SO(10) RG NOT futile : modest, calculable,constraining.
- MSGUT Parameter tomogram.
- RG analysis with EWRSB, m_F, M_{Soft}, M' ??
- Emerging Coherence :

  * $M_{q,l,\nu}, \theta_{q,l}$

  * B and L violation

  * Gauge Unification

  * R_p

  * SEESAW,

  * Leptogenisis

  * ASGUT(?) :

  "grail spoor" ??

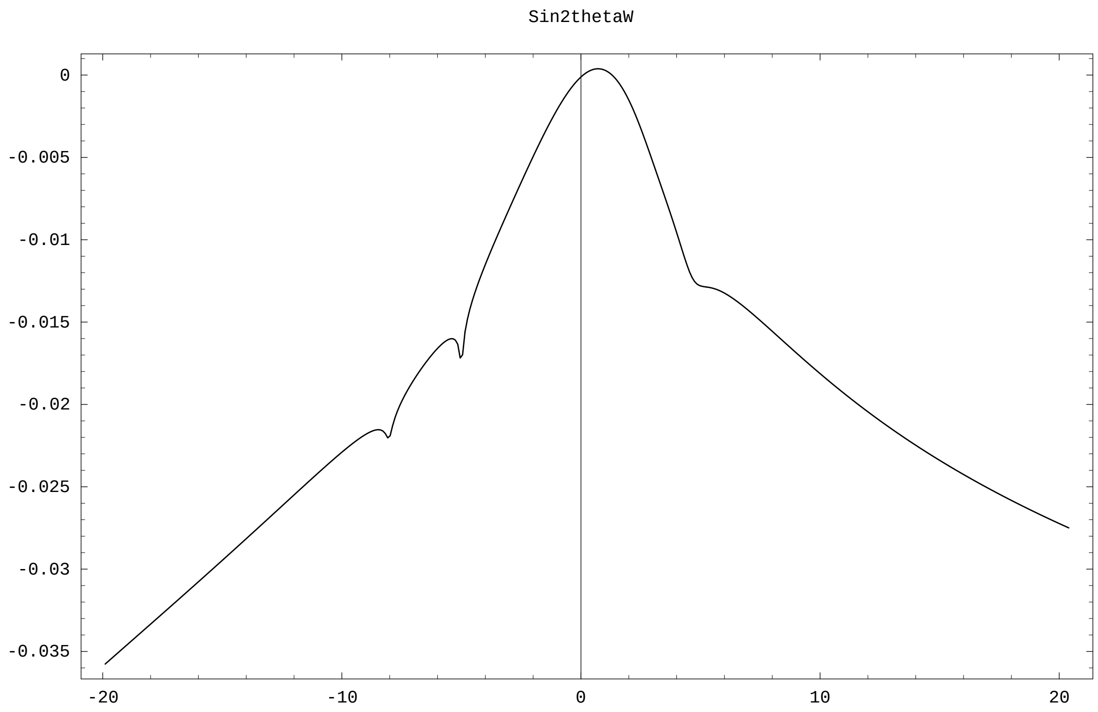

Figure 3: Plot of the threshold corrections to $Sin^2\theta_w$ vs ξ for real ξ : complex solution for x.

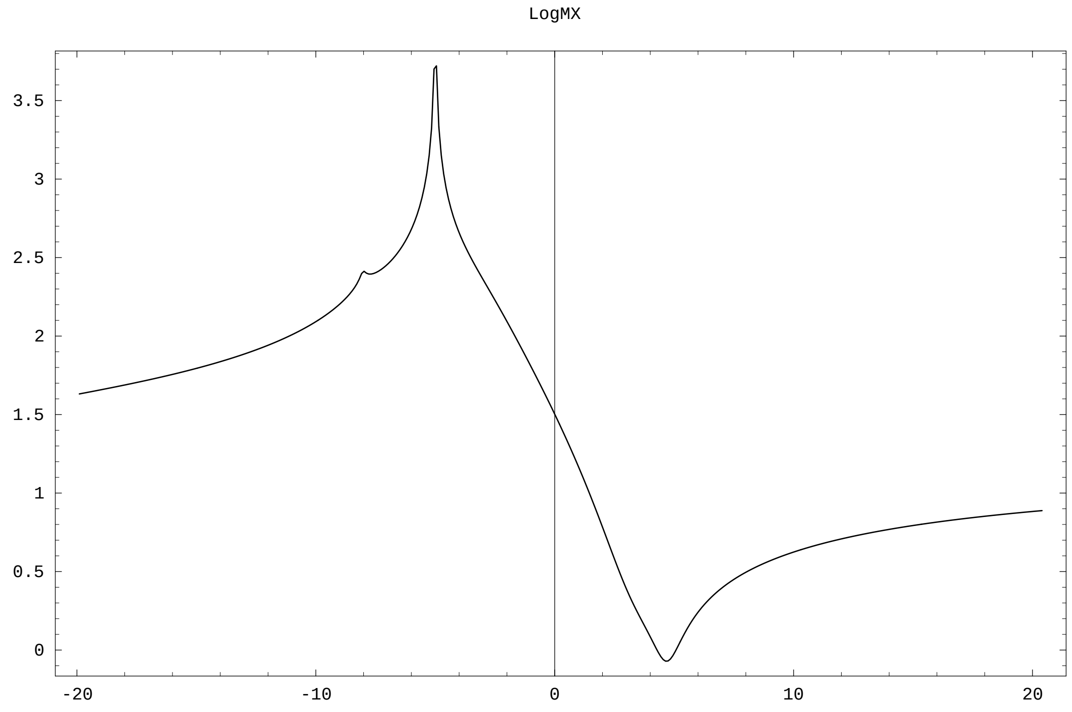

Figure 4: Plot of the threshold corrected $Log_{10} M_X/M_X^0$ vs ξ for real ξ : complex solution for x.

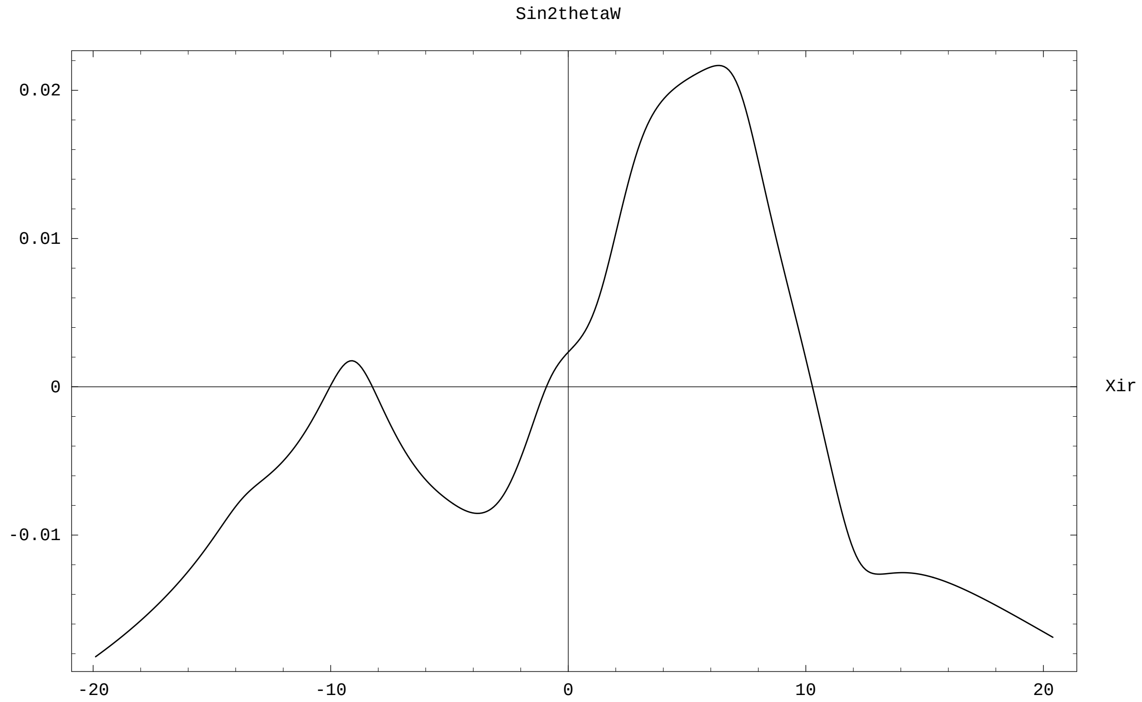

Figure 5: Plot of the threshold corrections to $Sin^2\theta_w$ vs $Re(\xi)$ for complex ξ : $Im\xi = 1.2$, first solution for x.

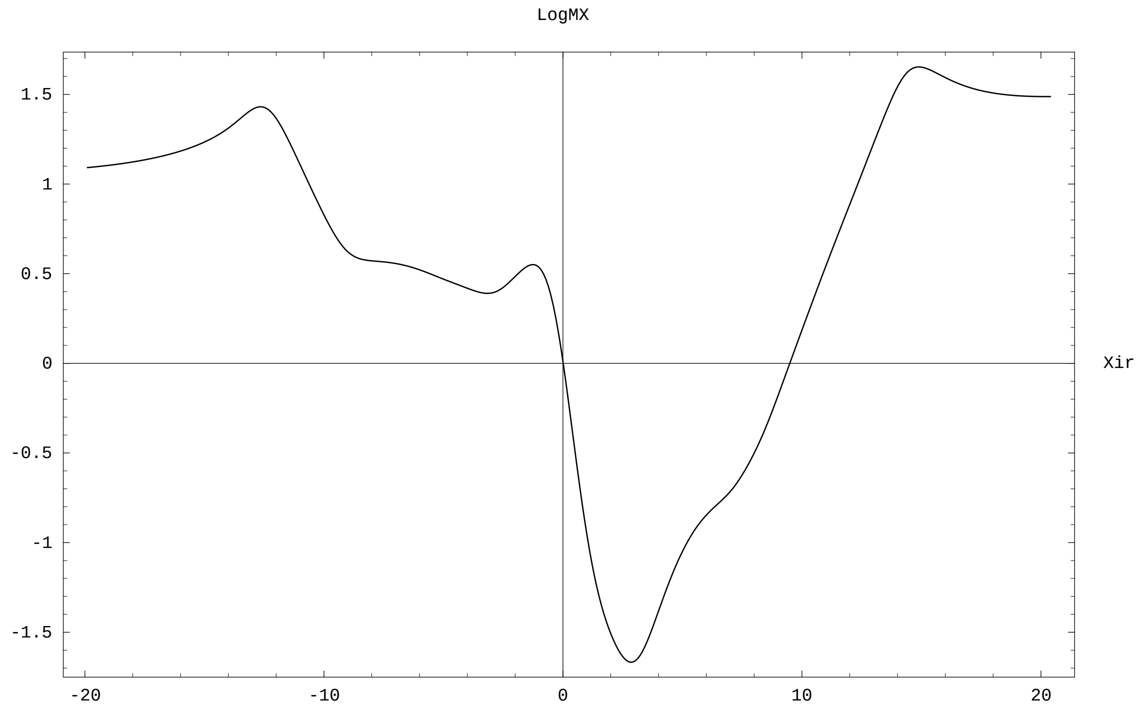

Figure 6: Plot of the threshold corrected $Log_{10}M_X/M_X^0$ vs $Re(\xi)$ for complex ξ : $Im\xi = 1.2$, first solution for x.