Center Vortices and the Gribov Horizon

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Cast of characters in three different confinement scenarios:

- Center Vortices
- Monopoles
- the Gribov horizon

The first two are related. A center vortex , in abelian projection, is a monopole-antimonopole chain.

Del Debbio, Faber, Olejnik & JG // Ambjorn, Giedt, J.G. // de Forcrand and Pepe // Polikarpov, Zakharov et al. // Chernodub et al. // Engelhardt

What about the Gribov horizon scenario?

1. Is it right?
2. Is it related to other ideas?
3. What ***is*** the Gribov horizon, anyway?

Content of this talk:

1) Yes; and 2) yes; and 3) I will explain...

In a confining theory, the energy of an isolated charge is infinite.

Why?

A warm-up exercise: the Coulomb self-energy of a static charge. Start with

$$\begin{aligned} H &= \int d^3x \; (E^2 + B^2) \; + \; H_{coul} \\ H_{coul} &= \int d^3x d^3y \;\; \rho(x) K(x,y) \rho(y) \\ K(x,y) &= M^{-1}(-\nabla^2) M^{-1} \\ \mathcal{E} &= e^2 K(x,x) \end{aligned}$$

where $M = -\nabla^2$,

$\mathcal{E}$ is the self-energy of a static charge at point x.

M is the **Faddeev-Popov** operator for the abelian theory. Let A^{θ}_{k} be an infinitesmal gauge transformation of A_k. Then

$$M_{xy} = \frac{\delta}{\delta\theta(x)} \nabla \cdot A^{\theta}(y) = -\nabla^2 \delta(x-y)$$

The eigenstates

$$M\phi^{(n)} = \lambda_n \phi^{(n)}$$

are just the plane wave states, with $\lambda_n = k_n^2$. On the lattice these states are discrete, and we can write the Green's function

$$G_{xy} = \left[M^{-1}\right]_{xy} = \sum_n \frac{\phi_x^{(n)} \phi_y^{(n)*}}{\lambda_n}$$

Then after a few manipulations

$$\mathcal{E} = e^2[M^{-1}(-\nabla^2)M^{-1}]_{xx}$$

$$= \frac{e^2}{L^3}\sum_n \frac{F(\lambda_n)}{\lambda_n^2}$$

where

$$F(\lambda_n) = (\phi^{(n)}|(-\nabla^2)|\phi^{(n)})$$

Let $\rho(\lambda)$ denote the normalized density of eigenvalues. Then at large volumes we can approximate the sum over eigenstates by an integral

$$\mathcal{E} = e^2 \int d\lambda\ \frac{\rho(\lambda)F(\lambda)}{\lambda^2}$$

In QED, its easy to show that

$$\rho(\lambda) = \frac{\sqrt{\lambda}}{4\pi^2} \quad , \quad F(\lambda) = \lambda$$

and also $\lambda_{min} \sim 1/L^2$, $\lambda_{max} \sim 1/a^2$, so that putting it all together

$$\begin{aligned} \mathcal{E} &= e^2 \int d\lambda \, \frac{\rho(\lambda)F(\lambda)}{\lambda^2} \\ &\sim e^2 \left(\frac{1}{a} - \frac{1}{L} \right) \end{aligned}$$

which is finite, at finite UV cutoff ***a***, as $L \to \infty$. But IR finiteness clearly depends on the small λ behavior of $\rho(\lambda)$ $F(\lambda)$. If instead

$$\lim_{\lambda \to 0} \frac{\rho(\lambda)F(\lambda)}{\lambda} > 0$$

then the Coulomb energy would be IR infinite.

In non-abelian theories, there are many gauge copies - Gribov copies - that satisfy the Coulomb gauge condition. The Gribov Region is the space of all Gribov copies with positive F-P eigenvalues.

The boundary of the Gribov region is the ***Gribov Horizon***. Configurations of the Gribov Horizon have at least one F-P eigenvalue $\lambda=0$.

Typical Coulomb-gauge lattice configurations are expected to approach the Gribov horizon in the infinite-volume limit.

But what counts for confinement is the density of eigenvalues $\rho(\lambda)$ near $\lambda=0$, and the “smoothness” of these near-zero eigenvalues, as measured by $F(\lambda)$.

Coulomb Self Energy – Yang-Mills

In Yang-Mills theory Faddeev-Popov operator depends on the gauge field

$$M[A] = \nabla \cdot \mathcal{D}[A]$$

The self-energy of an isolated static charge in color group rep. r, Casimir C_r, is $\mathcal{E}_r = g^2 C_r \mathcal{E}$, where

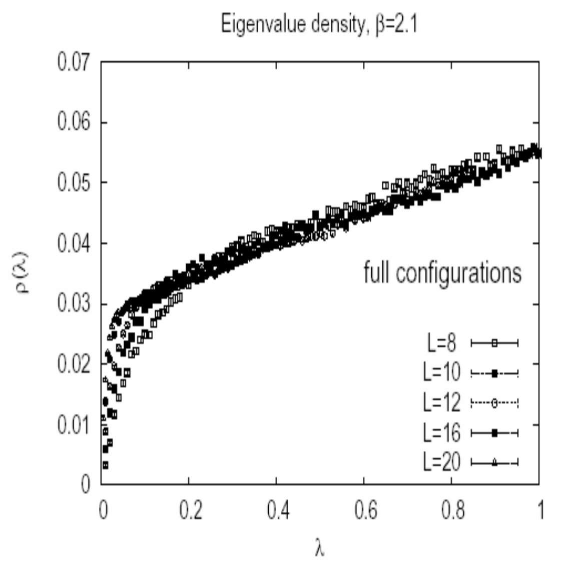
$$\mathcal{E} = \int d\lambda \; \frac{\rho(\lambda) F(\lambda)}{\lambda^2}$$

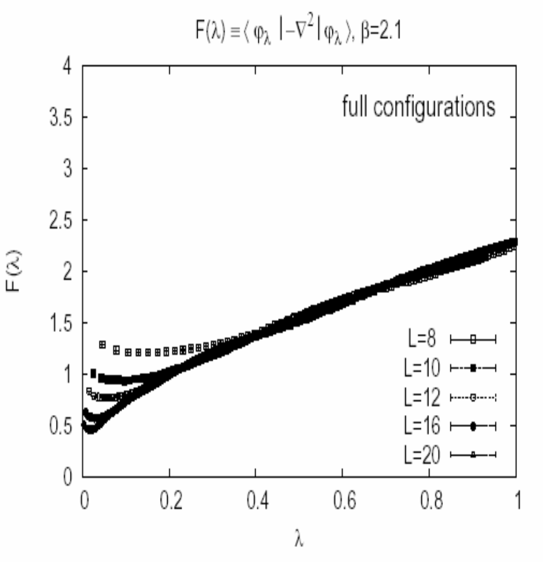
$$F(\lambda_n) = (\phi^{(n)} | (-\nabla^2) | \phi^{(n)})$$

We calculate $\rho(\lambda)$, $F(\lambda)$ numerically, on finite-size lattices, and extrapolate to infinite volume.

Method:

- gauge fields from lattice Monte Carlo, fix to Coulomb gauge
- find the first 200 eigenstates of the lattice Faddeev-Popov operator on each time-slice of each lattice configuration (Arnoldi algorithm)
- calculate $\rho(\lambda)$, $F(\lambda)$. Results, L=8 – 20:





From scaling of the distributions at small λ with L, we estimate at infinite volume

$$\rho(\lambda) \sim \lambda^{0.25} \quad , \quad F(\lambda) \sim \lambda^{0.4}$$

which implies

$$\mathcal{E} = \int d\lambda \, \frac{\rho(\lambda) F(\lambda)}{\lambda^2} \rightarrow \infty$$

in the infrared

(i.e. confinement)

Center Vortices

Center vortices are surfacelike (D=4) objects in the QCD vacuum which can be topologically linked to closed loops. Creation of a center vortex linked to a Wilson loop multiplies the loop by a center element of the gauge group.

The center vortex theory of confinement holds that the area law falloff of a Wilson loops is due to vacuum fluctuations in the number of vortices linking the loop.

In 1997, methods were devised for located center vortices in lattice configurations, and also for removing them.

There has since been ***lots*** of work on vortices in the lattice community.

Reviews: J.G., hep-lat/0301023,

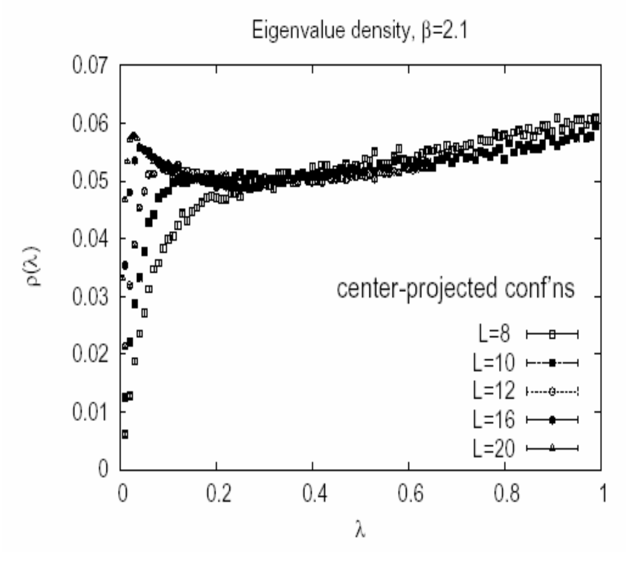
Engelhardt, plenary talk at Lattice 2004

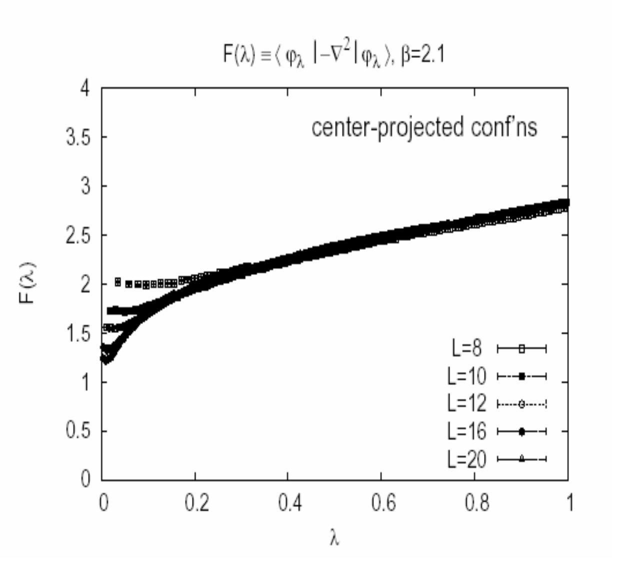
Numerical Findings:

- The sign of a large Wilson loop is strongly correlated with the vortex linking number;
- Vortex density scales according to asymptotic freedom;
- SU(2) action at vortex locations is much larger than the vacuum average;
- Center vortices account for the entire string tension.
- Removing center vortices:
  - removes the string tension
  - eliminates chiral symmetry breaking
  - takes the topological charge to zero

Using standard methods, we can decompose any lattice configuration into vortex-only and vortex-removed configurations.

Here is the result for the vortex-only configurations





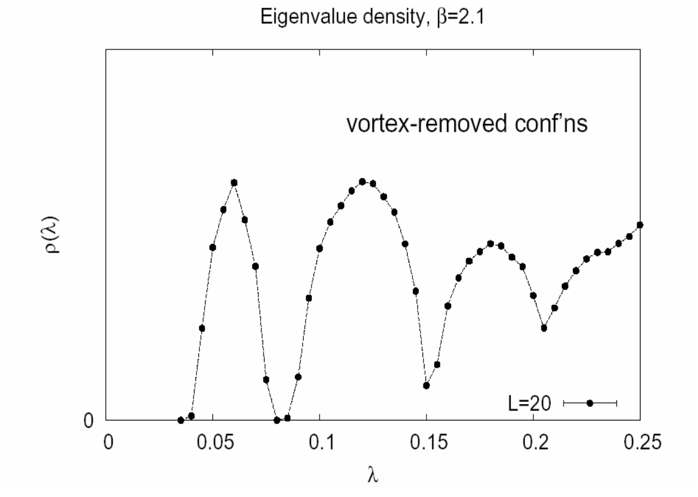
As before, we estimate

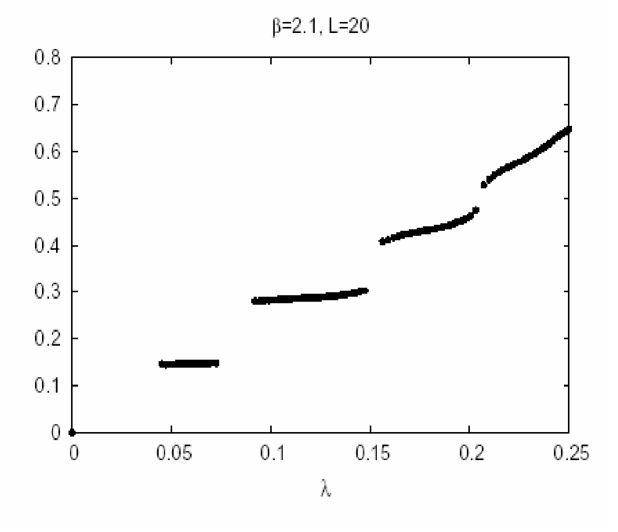
$$\rho(\lambda) \sim \lambda^{0\pm 0.05} \quad , \quad F(\lambda) \approx 1$$

And again

$$\mathcal{E} = \int d\lambda \, \frac{\rho(\lambda) F(\lambda)}{\lambda^2} \rightarrow \infty$$ **(confinement)**

Here is the result for the no-vortex configurations





"peaks" "bands"

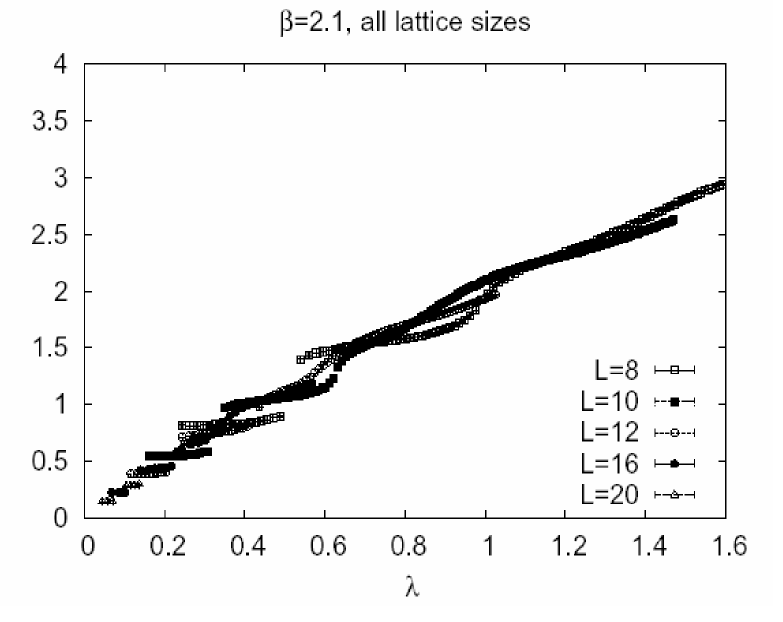
The number of eigenvalues in each "peak" of $\rho(\lambda)$, and each "band" of $F(\lambda)$, matches the degeneracy of eigenvalues of $-\nabla^2$, the zeroth-order Faddeev-Popov operator, at the given lattice size.

$\rho(\lambda)$ for the $-\nabla^2$ operator is just a series of δ-function peaks. In the vortex-only configurations, these peaks broaden to finite width, but the qualitative features of $\rho(\lambda)$ $F(\lambda)$ at zeroth order - no confinement - remain.

Further evidence: the low-lying eigenvalues scale with L as

$$\lambda_n \sim \frac{1}{L^2}$$

just like in the abelian theory, and looking at F(λ) at all lattice volumes



it seems that F(λ) » λ, again as in the abelian theory.

Some Analytical Results

Facts about vortices and the Gribov horizon, stated here without proof:

- Vortex-only configurations have non-trivial Faddeev-Popov zero modes, and therefore lie precisely on the Gribov horizon.

- The Gribov horizon is a convex manifold in the space of gauge fields, both in the continuum and on the lattice. The Gribov region, bounded by that manifold, is compact.

- Vortex-only configurations are conical singularities on the Gribov horizon.

Center vortices appear to have a special geometrical status in Coulomb gauge. The physical implications of this fact are not yet understood.

Conclusions

- The Coulomb self-energy of a color non-singlet state is infrared divergent, due to the enhanced density $\rho(\lambda)$ of Faddeev-Popov eigenvalues near $\lambda=0$.

This supports the Gribov-Zwanziger picture of confinement.

- The confining property of the F-P eigenvalue density can be entirely attributed to center vortices:

1. Enhancement of $\rho(\lambda)$ is found in vortex-only configurations.
2. The confining properties of $\rho(\lambda)$, $F(\lambda)$ disappear whenever vortices are either removed from lattice configurations, or - as in the Higgs phase - they cease to percolate.

These results establish a connection between the center vortex and Gribov horizon scenarios for confinement.